Projectile motion can be analysed quantitatively by treating the horizontal and vertical components of the motion independently.

- Construct, identify, and label displacement, velocity, and acceleration vectors.
- Use vector addition and subtraction to calculate net vector quantities.
- Resolve velocity into vertical and horizontal components, using $v_H = v\cos\theta$ and $v_v = v\sin\theta$ for the horizontal and vertical components respectively.
- Determine the velocity at any point, using trigonometric calculations or a scale diagram.

A **projectile** is any object moving under the influence of gravity only. Objects that are dropped or projected into the air near the surface of Earth are examples of projectiles. The motion of a projectile follows a curved path called a **parabola** near the surface of Earth. Figure 1.01 shows two examples of projectile motion.

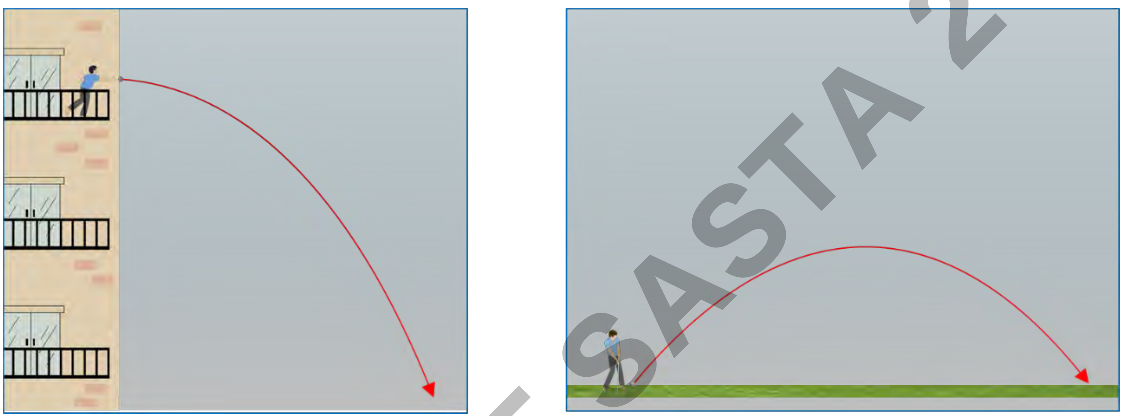


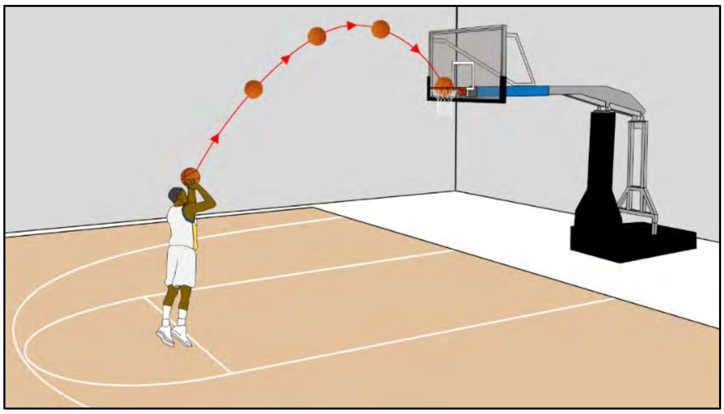
Figure 1.01: Examples of projectile motion.

Horizontal and vertical components of the motion

A projectile has motion in both the horizontal and vertical direction and each component of motion is analysed independently. The velocity of a projectile can be resolved into its horizontal and vertical components and the magnitude of each component is calculated using trigonometry.

Example

A basketballer shooting a free-throw releases the ball with an initial velocity of 7.1 m.s^{-1} at an angle of 54^{o} above the horizontal.

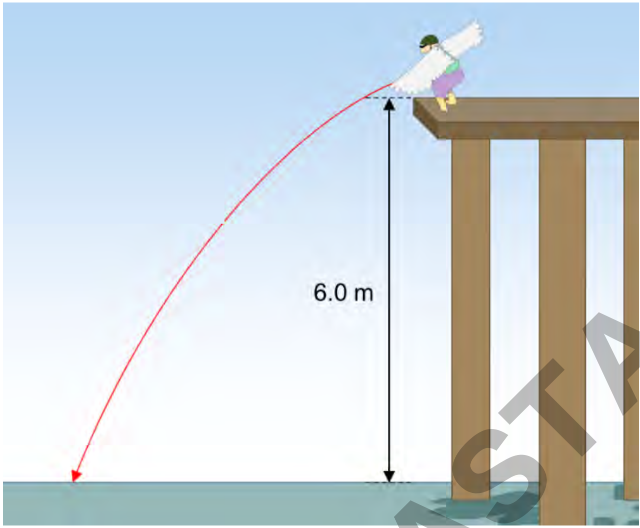


Question 5

The Birdman Rally is a fundraising competition in which contestants construct home-made gliders and leap from a jetty.

The first South Australian competition was held in April 1975 on the River Torrens.

Contestants ran horizontally before leaping from a platform that was 6.0 m above the water surface.



(a) Show that the time of flight of a contestant leaping from the platform was approximately 1.1 s.

Assume air resistance is negligible and that the contestant was in free fall.

(2 marks) **KA4**

(b) One contestant in 1975 achieved a horizontal range of 4.5 m in free fall.

Show that the winning contestant had a horizontal component of velocity equal to 4.1 $m.s^{-1}$.

(2 marks) **KA4**

(c) State and explain the magnitude of the horizontal component of velocity of the contestant at the instant they contacted the water surface.

____

____ (2 marks) **KA1**

(d) State why the magnitude of the vertical component of the initial velocity of a contestant was zero.

(1 mark) **KA1**

(e) Show that the vertical component of velocity of the contestant in free fall was approximately 11 $m.s^{-1}$ at the instant they contacted the water surface.

(2 marks) **KA4**

(f) Calculate the magnitude and direction of the resultant velocity of the contestant in free fall at the instant they contacted the water surface.

Draw a fully labelled vector diagram to support your answer.

Space for vector diagram

(6 marks) **KA4**

(g) State and explain one method that could be used by the contestant to increase their horizontal range.

(2 marks) **KA2**

The motion formulae are used to calculate measurable quantities for objects undergoing projectile motion.

- Explain qualitatively that the maximum range occurs at a launch angle of 45^0 for projectiles that land at the same height from which they were launched.
- Describe the relationship between launch angles that result in the same range.
- Describe and explain the effect of launch height, speed, and angle on the time of flight and the maximum range of a projectile.

The maximum range (s_H) of a projectile that lands at the same height from which it was launched is affected by the launch angle. The table below shows the effect of launch angle on the horizontal range of a projectile launched with an initial velocity of 25 $m.s^{-1}$.

Launch angle (°)	Range (m)
0	0
10	21.79
20	40.97
30	55.19
40	62.76
45	63.73
50	62.76
60	55.19
70	40.97
80	21.79
90	0

Two conclusions are drawn from the data in the table.

(1) The maximum range of the projectile occurs when the launch angle is 45^0 (Figure 1.06).

(2) The horizontal range is the same for two launch angles that sum to 90^0. For example: the horizontal range is the 55.19 m for launch angles of 30 and 60 degrees (30 + 60 = 90).

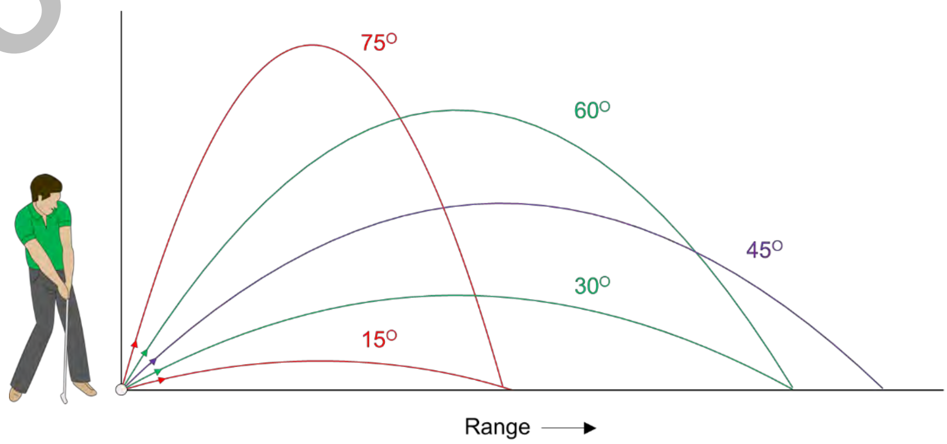
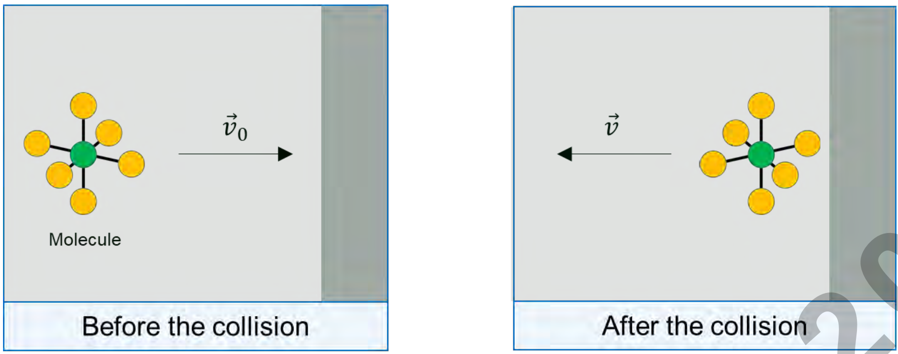


Figure 1.06: Effect of launch angle on range (uni-level projection).

Question 15

A molecule collides elastically with the walls of a gas cylinder.

The molecule has a mass of 2.45 x 10^{-25} kg and an initial velocity of 220 $m.s^{-1}$ at right angles towards the wall.



(a) Calculate the initial momentum of the molecule.

(2 marks) **KA4**

(b) The molecule moves away from the wall in the opposite direction with no change in velocity.

Show that the change in momentum is approximately 1.1 x 10^{-22} $kg.m.s^{-1}$.

State the direction of the change in momentum.

(3 marks) **KA4**

(c) The molecule is in contact with the wall of the container for 1.0 x 10^{-9} s.

(1) Derive $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ using Newton's Second Law of Motion.

(2 marks) **KA1**

(2) Calculate the force applied by the molecule on its container.

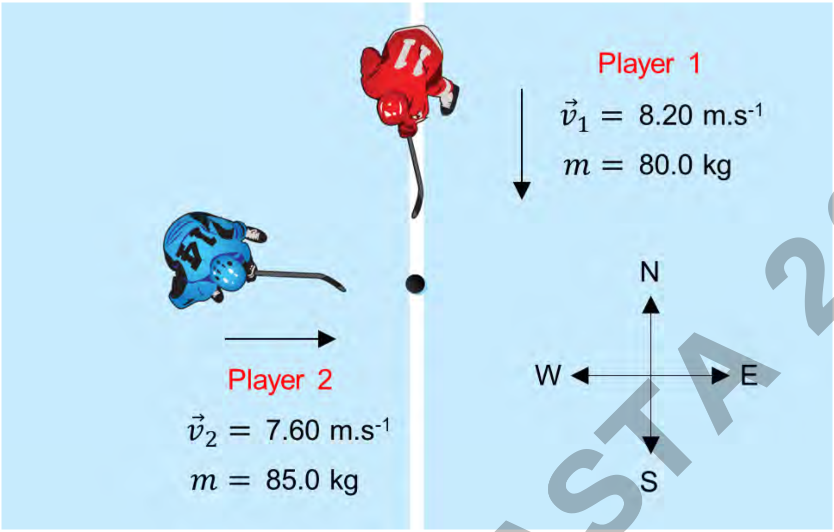
(2 marks) **KA4**

Question 22

Two ice-hockey players collide when competing for the puck.

The two players move together after the collision.

The mass and initial velocity of the two players is provided in the diagram below.



(a) Calculate the magnitude and state the direction of the momentum of the two players after the collision.

Construct a fully labelled vector diagram to support your answer.

Space for vector diagram

(6 marks) **KA4**

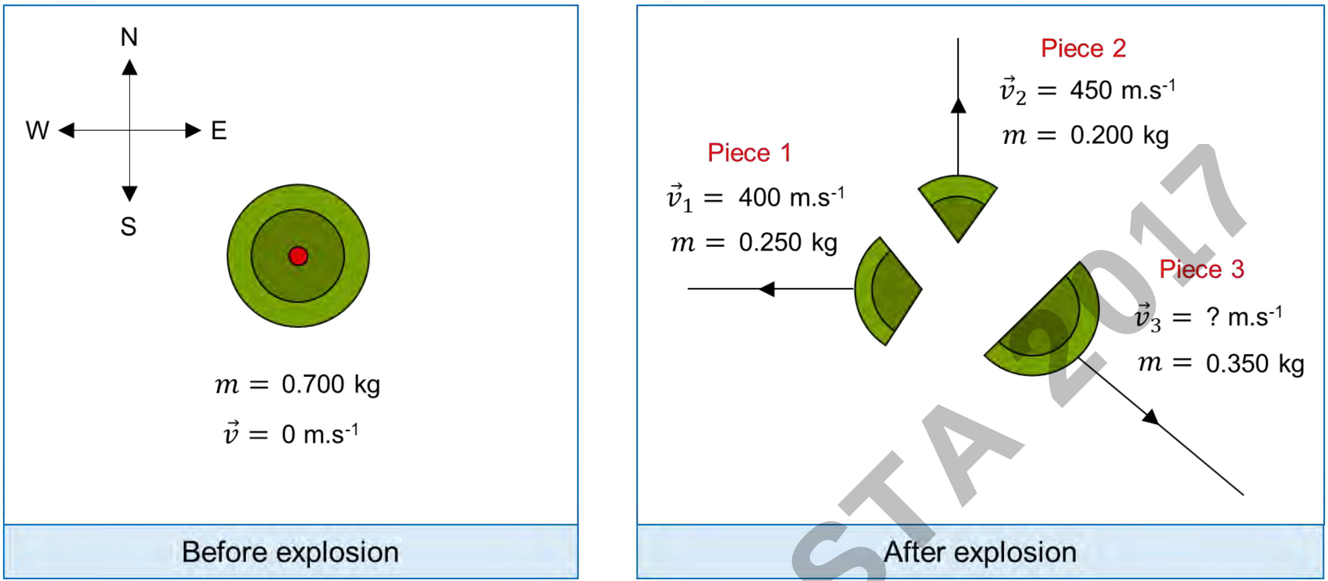
(a) Calculate the final velocity of the two players.

(2 marks) **KA4**

Question 23

A land mine explodes into three pieces with different masses.

The mass and velocity of each piece is provided in the diagram below.



(a) Calculate the magnitude and state the direction of the momentum of piece 3.

Construct a fully labelled vector diagram to support your answer.

Space for vector diagram

(6 marks) **KA4**

(b) Calculate the final velocity of piece 3.

(2 marks) **KA4**

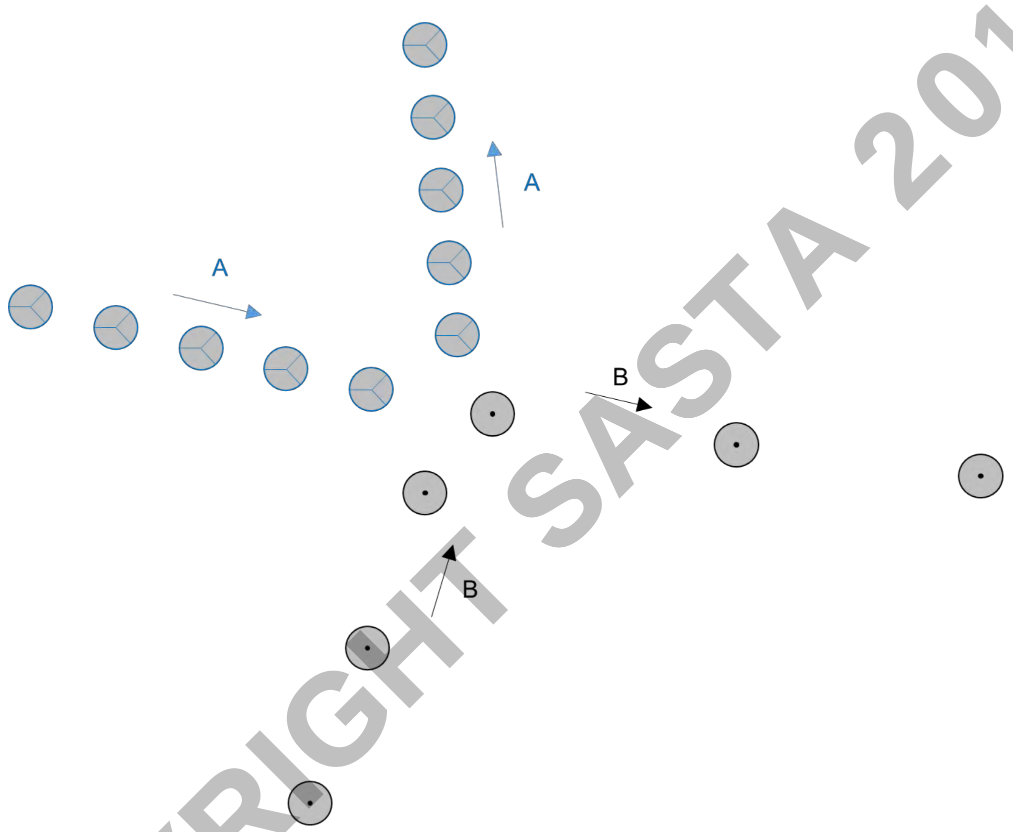
Question 24

Multi-image representations can be used to determine whether momentum was conserved in a collision or explosion.

Draw vector diagrams and determine whether momentum was conserved in the collisions below.

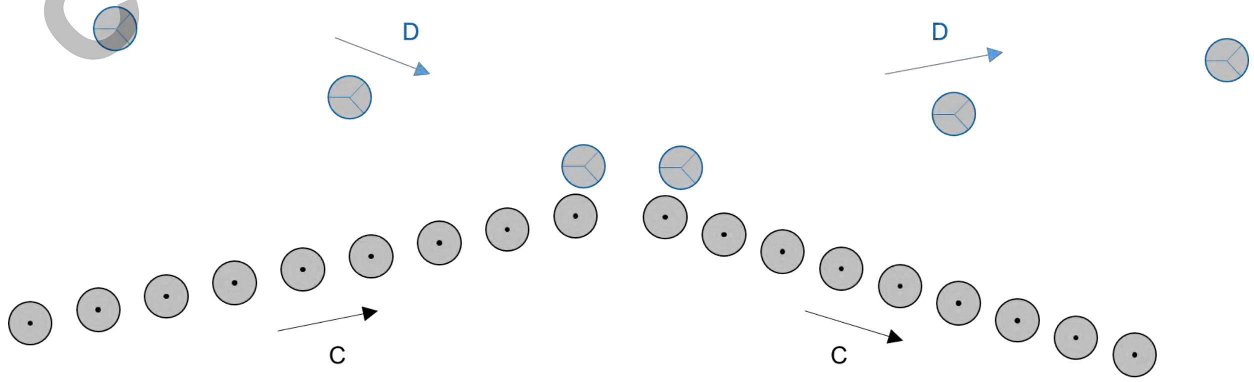
(a) Particles A and B collide on a frictionless surface.

The mass of particle A is twice the mass of particle B. (3 marks) **KA4**



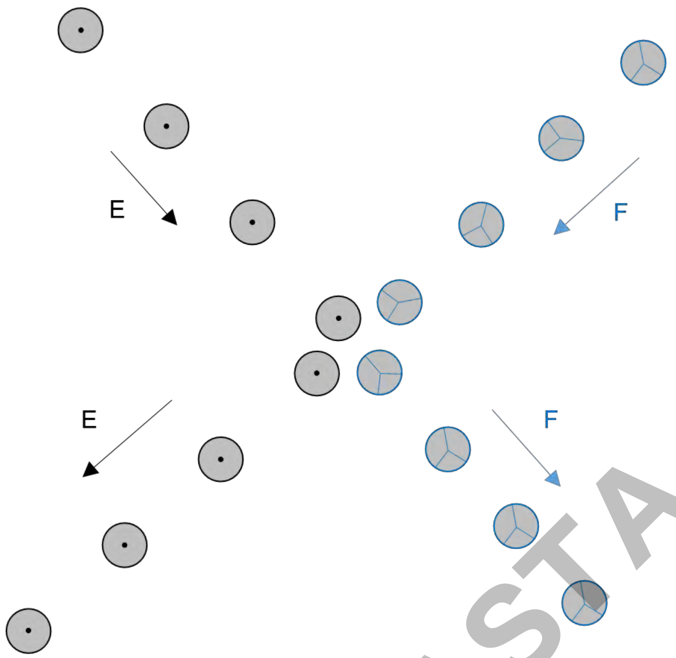
(b) Particles C and D collide on a frictionless surface.

The mass of particle C is four times the mass of particle D. (3 marks) **KA4**



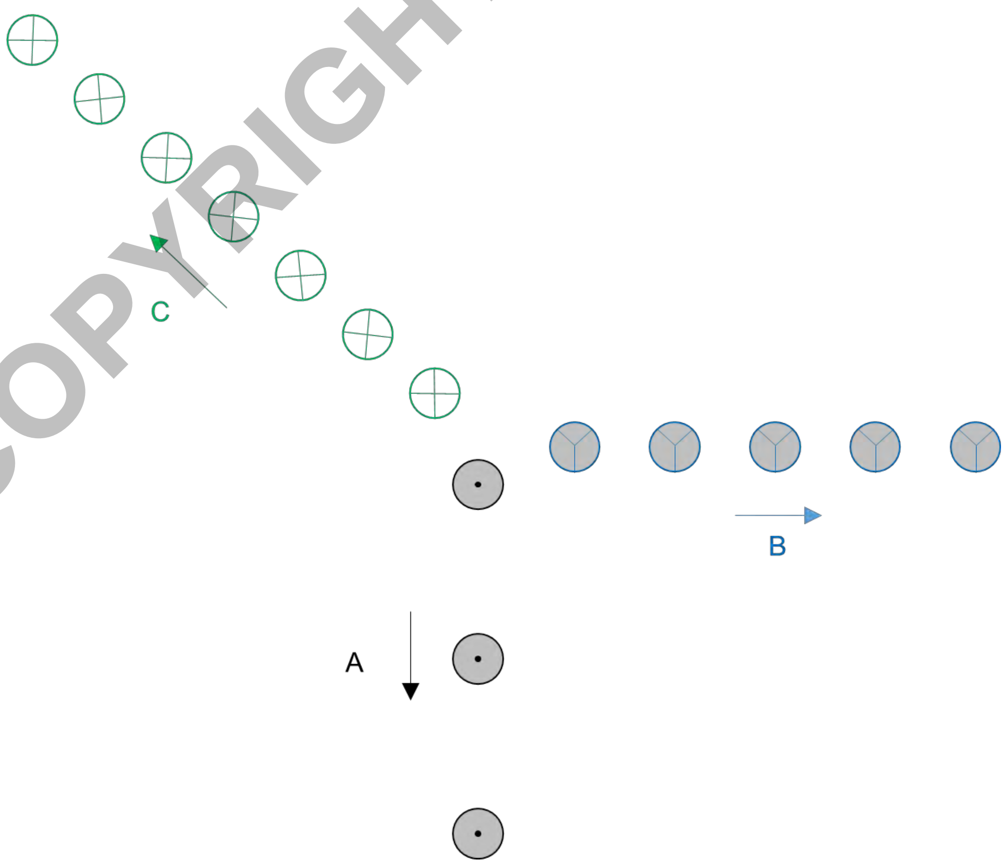
(c) Particles E and F have the same mass.

Determine whether the collision below occurred on a frictionless surface. (3 marks) **KA4**



(d) A stationary mass explodes into 3 separate particles with different masses.

The masses of particles A, B and C are m, $2m$ and $3m$ respectively. (3 marks) **KA4**



Ion propulsion spacecraft

Ion thrusters are a type of spacecraft propulsion system that uses electrical energy to accelerate ions to the rear of a spacecraft (Figure 1.18).

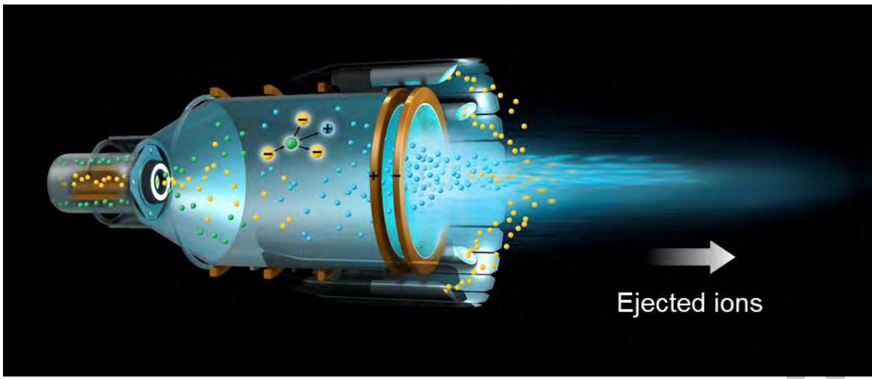


Figure 1.18: Illustration of an ion thruster.

The ions gain momentum to the rear of the spacecraft which causes the spacecraft to gain an equal magnitude of momentum in the opposite direction. Ion propulsion systems produce a much smaller force relative to chemical rocket thrusters, but the force acts over a longer time interval (in excess of 10 000 hours) which allows the spacecraft to accelerate slowly to a very high final velocity (around 15 000 $m.s^{-1}$).

Solar sail

A solar sail is a spacecraft propulsion system that uses sunlight to accelerate a spacecraft to a high velocity. A solar sail is composed a very thin material with a large surface area that is deployed from a spacecraft in a similar manner to a sail on a boat (Figure 1.19).

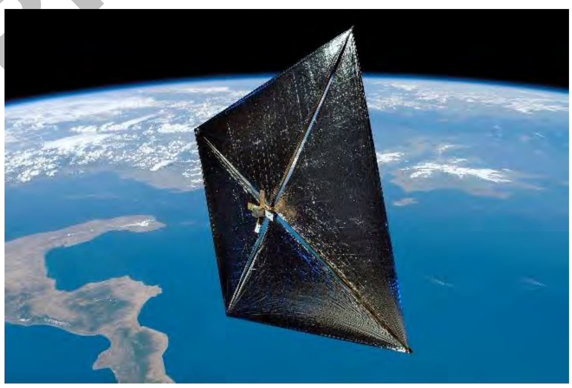


Figure 1.19: Illustration of a solar sail attached to a satellite (spacecraft).

Sunlight is composed of small bundles (quanta) of energy called **photons**. Photons have momentum which is transferred to a spacecraft when photons are absorbed or reflected from the solar sail. Each photon transfers a small amount of momentum to the solar sail which causes the spacecraft to accelerate slowly to a very high final velocity. The magnitude of the change in momentum of the spacecraft is dependent on whether the photons are absorbed or reflected from the solar sail.

Question 25

Dawn is a spacecraft launched by NASA in 2007 with the mission of studying two large protoplanets in the asteroid belt between Mars and Jupiter.

The Dawn spacecraft is propelled by xenon ion thrusters.

(a) Describe and explain how the emission of xenon ions increases the velocity of the Dawn spacecraft.

____

____

____

____

____

______________________________________ (3 marks) KA2

(b) At one stage of the mission, the emission of xenon ions increased the velocity of the Dawn spacecraft by 26.7 $m.s^{-1}$ over a time interval of 3.46×10^5 s.

The mass of the spacecraft over this time interval was 1.17×10^3 kg.

Calculate the average force exerted by the xenon ion thrusters during this time interval.

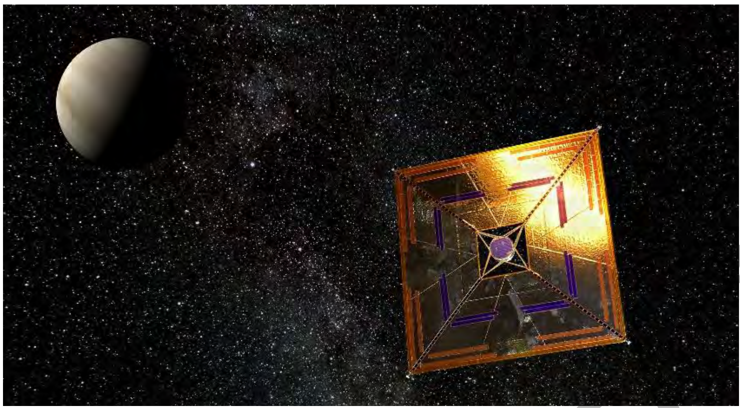
Assume that the decrease in mass of the Dawn spacecraft due to the expulsion of fuel was negligible during this time interval.

(4 marks) KA2

Question 26

IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun) was a spacecraft (m = 300 kg) built by the Japan Aerospace Exploration Agency (JAXA).

IKAROS was launched in 2010, and was the first spacecraft to successfully demonstrate solar sail technology as the main propulsion system in interplanetary space.



(a) Photons were reflected from the solar sail attached to IKAROS.

Describe and explain how the reflection of solar photons caused IKAROS to accelerate.

Draw a vector diagram to explain your answer.

(4 marks) **KA2**

(b) Solar photons exert a force of 9.08×10^{-6} N per square metre of solar sail.

The solar sail attached to IKAROS was 196 m^2.

Calculate the time taken for the velocity of IKAROS to increase by 400 $m.s^{-1}$.

Give the answer in units of Earth days (one Earth day is 86 400 s).

(5 marks) **KA2**

1.3: Circular Motion and Gravitation

An object moving in a circular path at a constant speed undergoes uniform circular motion. This object undergoes centripetal acceleration, which is directed towards the centre of the circle.

The magnitude of the centripetal acceleration is constant for a given speed and radius and given by $a = \frac{v^2}{r}$.

The formula $v = \frac{2\pi r}{r}$ relates the speed, v, to the period, T, for a fixed radius.

- Solve problems involving the use of the formulae $a = \frac{v^2}{r}$, $v = \frac{2\pi r}{r}$ and $F = ma$.
- Use vector subtraction to show that the change in the velocity, $\Delta\vec{v}$ and hence the acceleration, of an object over a very short time interval is directed towards the centre of the circular path.

Uniform circular motion refers to the motion of an object moving at a constant speed in a circular path. The velocity of an object in uniform circular motion is along the tangent to the circle at any point on the circular path, meaning that the direction of the velocity is at right angles to a line joining the object to the centre of the circular path (Figure 1.21).

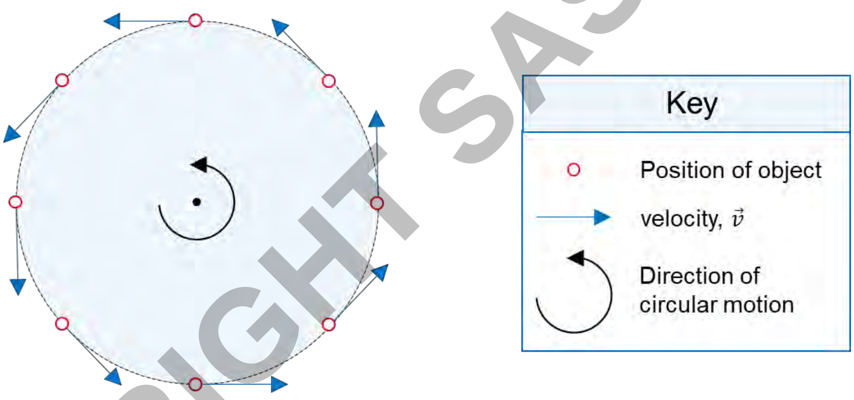


Figure 1.21: Direction of the velocity of an object in uniform circular motion.

Speed and period

The speed (v) of an object in uniform circular motion is equal to the distance travelled in one revolution divided by the period (T). The distance travelled by an object in one revolution is equal to the circumference of the circular path ($2\pi r$).

Formula	$v = \frac{2\pi r}{T}$

Symbol	Variable	SI unit
v	Speed of object	($m.s^{-1}$)
r	Radius of the circular path of the object	(m)
T	Period/ time taken for one revolution	(s)

Centripetal acceleration

The direction of motion of an object in uniform circular motion changes with time as shown in Figure 1.21. Any change in the magnitude or direction of velocity over time is acceleration. An object in uniform circular motion is accelerating as the direction of motion is changing with time. The acceleration of an object in uniform circular motion is called **centripetal acceleration** as the acceleration is directed towards the centre of the circular path (centripetal means “centre-seeking”).

The magnitude of the centripetal acceleration is calculated using the formula below.

Formula	$a = \frac{v^2}{r}$

Symbol	Variable	SI unit
a	Centripetal acceleration	($m.s^{-2}$)
v	Velocity	($m.s^{-1}$)
r	Radius of the circular path of the object	(m)

Vector subtraction is used to show that acceleration of an object in uniform circular motion is directed towards the centre of the circular path. Figure 1.22 shows the initial ($\vec{v}_0$) and final velocity ($\vec{v}$) vectors of an object in uniform circular motion. The change in velocity ($\Delta\vec{v}$) between points A and B occurs over a very short time interval (Δt).

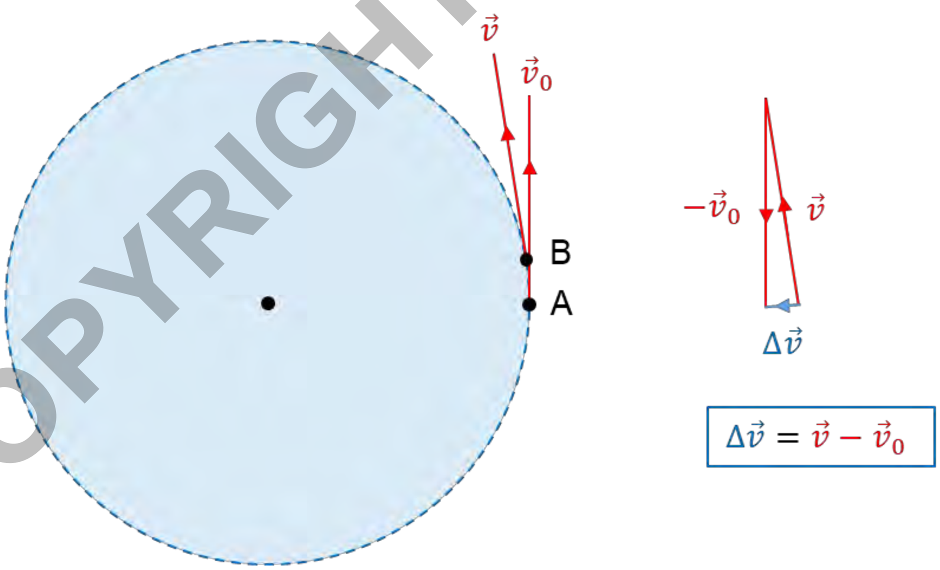


Figure 1.22: Vector subtraction showing the change in velocity of an object in uniform circular motion.

Figure 1.22 shows that the change in velocity ($\Delta\vec{v}$), and hence the acceleration, of an object is directed towards the centre of the circular path. This is only the case when the change in velocity is measured over a short time interval such as the interval shown in Figure 1.22.

Force causing centripetal acceleration

An object in uniform circular motion has centripetal acceleration which is constant in magnitude and is directed towards the centre of the circular path. Any object with constant acceleration is subject to a constant force that acts in the direction of the acceleration. The force causing centripetal acceleration is perpendicular to the velocity of the object and is directed towards the centre of the circular path (Figure 1.23).

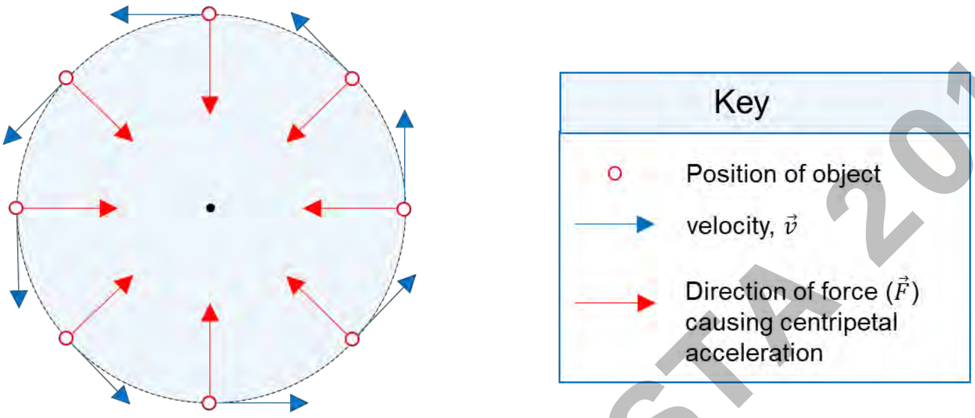
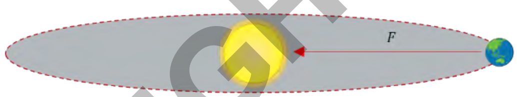
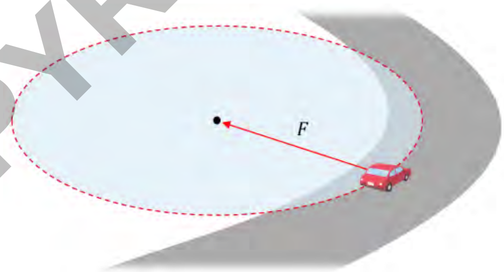
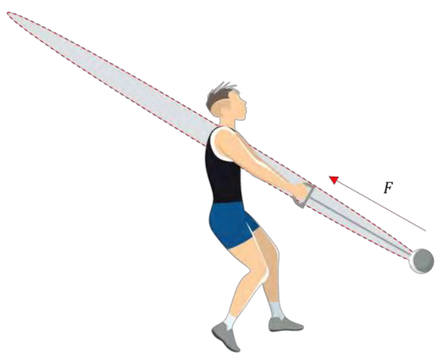


Figure 1.23: Direction of the force causing centripetal acceleration.

Some examples of forces causing centripetal acceleration are identified below.

Example	Diagram	Description
Planetary motion		The gravitational force causes the centripetal acceleration of planets around stars such as the Earth around the sun.
Car turning a corner on a flat road		Friction between the tyres and the surface causes the centripetal acceleration of a car turning a corner on a flat road.
Hammer throw		Tension in the steel wire causes the centripetal acceleration of the metal ball before it is released by an athlete.

The magnitude of a force causing centripetal acceleration is calculated using the formula below.

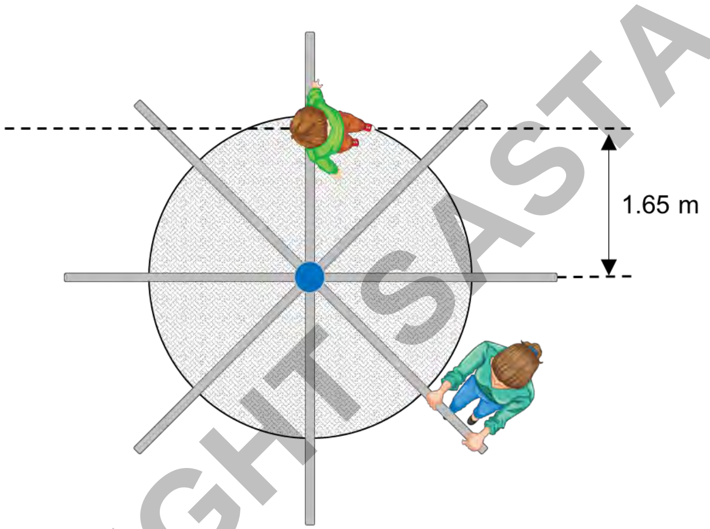
Formula	$F = ma = m\dfrac{v^2}{r}$

Symbol	Variable	SI unit
m	Mass of object	(kg)
v	Speed of object	($m.s^{-1}$)
r	Radius of the circular path of the object	(m)

Example:

A mother pushes her child (m = 20.0 kg) on a roundabout.

The force applied by the mother causes the child to move in a circular path of radius 1.65 m.



The period of one revolution of the child was 3.00 seconds whilst in uniform circular motion.

Calculate the force causing the centripetal acceleration of the child whilst they were in uniform circular motion.

v	=	$\dfrac{2\pi r}{T}$
v	=	$\dfrac{2\pi(1.65)}{3.00}$
v	=	$3.46\ m.s^{-1}$
F	=	$m\dfrac{v^2}{r}$
F	=	$\dfrac{20.0 \times (3.46)^2}{1.65}$
F	=	145 N

Question 27

The Hubble Space Telescope (HST) is a space telescope that was launched into orbit around the Earth in 1990 and remains in operation.

(a) The force causing the centripetal acceleration of the HST acts along a line between the centres of the Earth and the HST.

The radius of the orbit of the HST is 6.99×10^6 m.

(1) Identify the force causing the centripetal acceleration of the HST as it orbits Earth.

____ (1 mark) **KA2**

(2) The magnitude of the force causing the centripetal acceleration is 8.97×10^4 N.

The mass of the HST is approximately 1.10×10^4 kg.

Show that the speed of the HST is approximately 7.55×10^3 m.s^{-1}.

(2 marks) **KA4**

(3) Calculate the time taken for the HST to complete one full revolution around the Earth.

Give your answer in units of minutes (1 min = 60 s).

(3 marks) **KA2**

(b) The radius of Earth is 6.37×10^6 m.

Determine the altitude (h) of the HST above the surface of the Earth.

Give your answer in kilometres (km).

(3 marks) **KA2**

Normal force

Any vehicle at rest or in motion on a flat surface experiences a normal force that is perpendicular to the surface. The magnitude of the normal force is equal to the weight (mg) of the vehicle in situations where the vehicle is on a flat surface with no incline. However, in situations where the vehicle is on a flat surface that is inclined at an angle, the normal force has perpendicular components as shown in Figure 1.25.

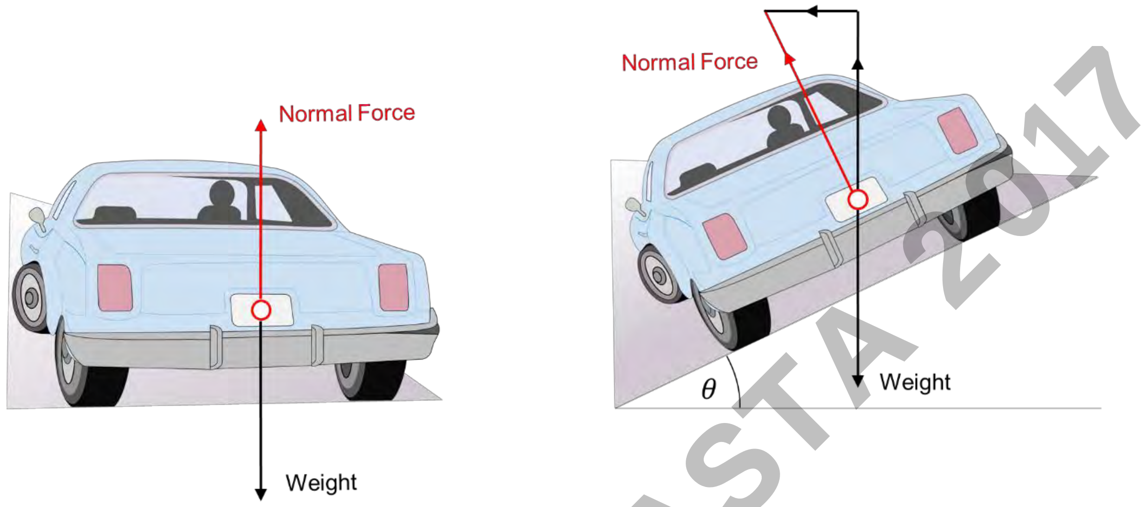


Figure 1.25: Direction of the normal force on a car moving on a flat road.

Banked curves

Roads and race tracks can be banked at an angle above the horizontal such that the outer edge of the road is higher than the inner edge. This is called **superelevation** and means that some or all of the centripetal acceleration of a vehicle is provided by the horizontal component of the normal force as shown in Figure 1.26.

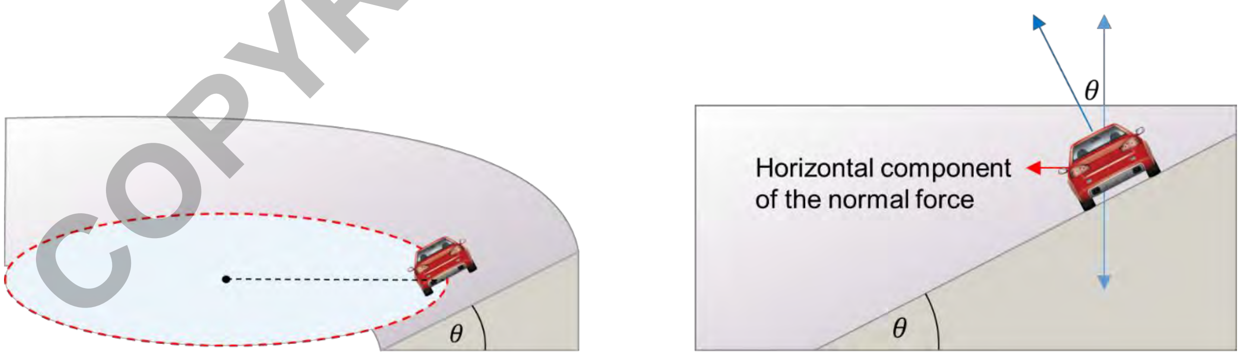


Figure 1.26: Car turning a corner on a banked road.

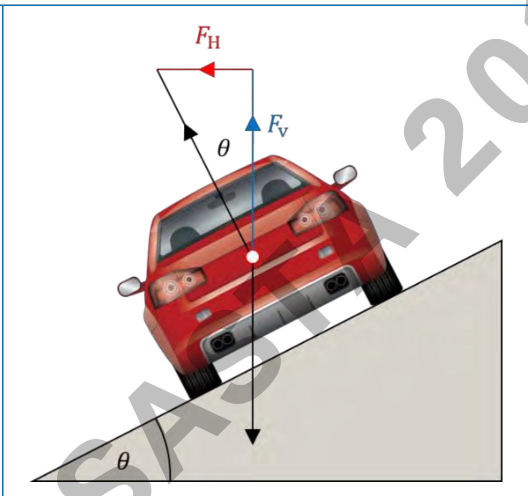
Figure 1.26 shows that the horizontal component of the normal force is directed towards the centre of the circular path as a vehicle turns on a road that is banked at an angle. The horizontal component of the normal force provides the centripetal acceleration which effectively reduces the reliance on the friction force to allow the vehicle to turn safely at a given speed.

Banked curve formula

Engineers construct curved roads and racetracks with banking angles that allow vehicles to travel safely at high speeds with no reliance on friction. The banking angles chosen by engineers are sufficient to ensure that the centripetal acceleration of a vehicle is provided solely by the horizontal component of the normal force.

A formula is derived that relates the banking angle (θ) of a curved road or racetrack to the speed of the vehicle (v) and the radius (r) of the circular path.

$$
\begin{aligned}
\tan\theta &= \frac{\text{opp}}{\text{adj}} \\
\tan\theta &= \frac{F_H}{F_v} \\
\tan\theta &= \frac{mv^2/r}{\text{mg}} \\
\tan\theta &= \frac{mv^2}{\text{mgr}} \\
\tan\theta &= \frac{v^2}{\text{gr}} \\
\theta &= \tan^{-1}\frac{v^2}{\text{gr}}
\end{aligned}
$$



Example:

The Indianapolis Motor Speedway is a car racing track located in Indiana in the United States of America. The race track has four identical turns that are banked at an angle of 9.0^{0} above the horizontal. Cars travel at high speeds around the turns which have a radius of 250 m.

Calculate the maximum speed of a racing car around these bends with no reliance on friction to cause the centripetal acceleration.

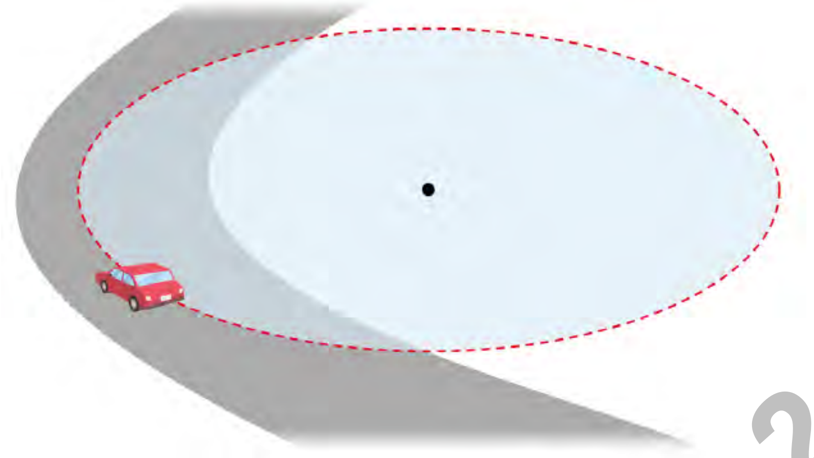
Notes

- The speed is calculated by rearranging the banking angle formula.

$$
\begin{aligned}
\tan\theta &= \frac{v^2}{\text{gr}} \\
v^2 &= \tan\theta\text{gr} \\
v &= \sqrt{\tan\theta\text{gr}} \\
v &= \sqrt{\tan 9.0 \text{ x } 9.8 \text{ x } 250} \\
v &= 20\ \text{m.s}^{-1}\ \ (19.7)
\end{aligned}
$$

Question 30

A car travels round a curve of radius 300 m along a flat road.



(a) Draw a vector on the diagram above to represent the direction of the force causing the centripetal acceleration of the car.

(1 mark) **KA4**

(b) The maximum frictional force between the tyres and the road is 70% of the weight (mg) of the car.

(1) Calculate the maximum speed at which the car can travel round the curve relying only on friction.

(3 marks) **KA2**

(2) Describe and explain the path taken by the car if it exceeds this speed.

____

____

____

______________________________________ (2 marks) **KA2**

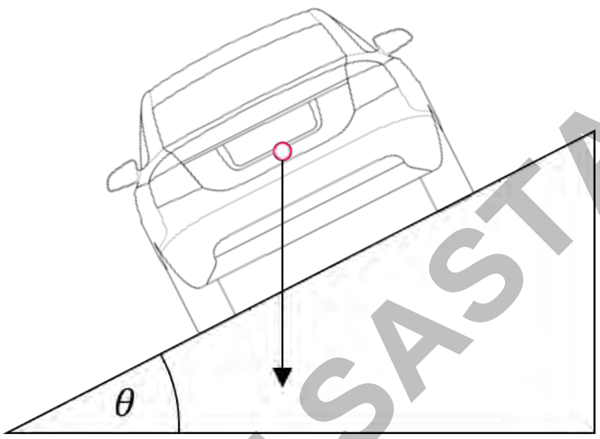
Question 31

Calder Park Raceway is a motor racing venue located near Melbourne, Australia.

The Thunderdome is a speedway located on the grounds of Calder Park Raceway.

The track inside the Thunderdome has four turns that are banked at the same angle.

(a) The diagram below shows a car turning a corner that is banked at an angle.



(1) Draw and label all force vectors acting on the car as it turns the corner.

(4 marks) **KA1**

(2) Derive the formula $\tan\theta = \frac{v^2}{rg}$ given the forces acting on the car as it turns the corner.

(3 marks) **KA1**

(b) The top speed on turns 1 and 2 (r = 210 m) is approximately 31 $m.s^{-1}$.

Calculate the banking angle for turns 1 and 2.

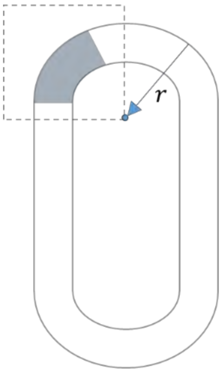
(2 marks) **KA2**

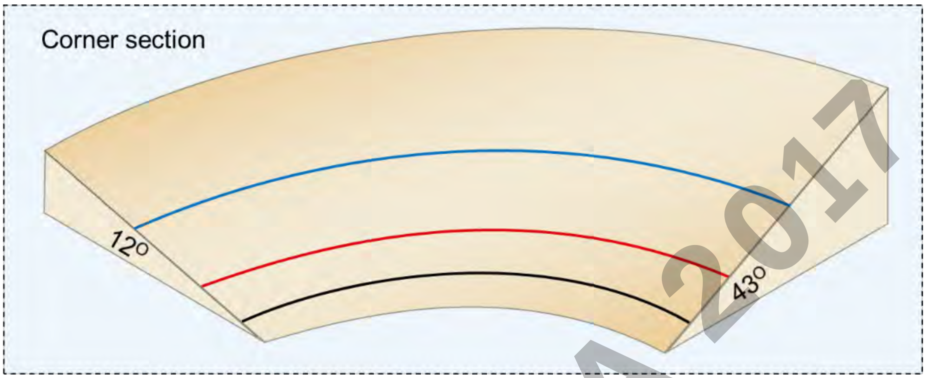
Question 33

Velodromes are arenas with tracks designed for high-speed bicycle races.

The Adelaide Super-Drome is a velodrome located in Adelaide, South Australia.

A diagram showing the corner section of the velodrome is given below.





The track is banked at an angle of 43^{o} at the corner section where the maximum speed is $12.3\ m.s^{-1}$.

(a) Show that the radius of the circular path that coincides with the corner section is 16.6 m.

(3 marks) **KA2**

(b) Calculate the magnitude and state the direction of the centripetal acceleration of a cyclist travelling at the maximum speed around the corner section of the track.

(3 marks) **KA2**

(c) Explain how a banked curve reduces the reliance on friction to provide centripetal acceleration for cyclists travelling around the track inside the velodrome.

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____

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____ (2 marks) **KA1**

All objects with mass attract one another with a gravitational force; the magnitude of this force can be calculated using Newton's law of universal gravitation. The force between two masses, m_1 and m_2 separated by distance, r, is given by $F = G\frac{m_1 m_2}{r^2}$

- Use Newton's Universal Law of Gravitation and the Second Law of Motion to calculate the value of the acceleration due to gravity, **g**, on a planet or moon.
- Solve problems using Newton's Universal Law of Gravitation.
- Use proportionality to discuss changes in the magnitude of the gravitational force on each of the masses as a result of a change in one or both of the masses and/or a change in the distance between them.
- Explain that gravitational forces are consistent with Newton's Third Law.

The **gravitational force** (F) is a mutual force of attraction between any two masses (m_1 and m_2) that are separated by a distance (r).

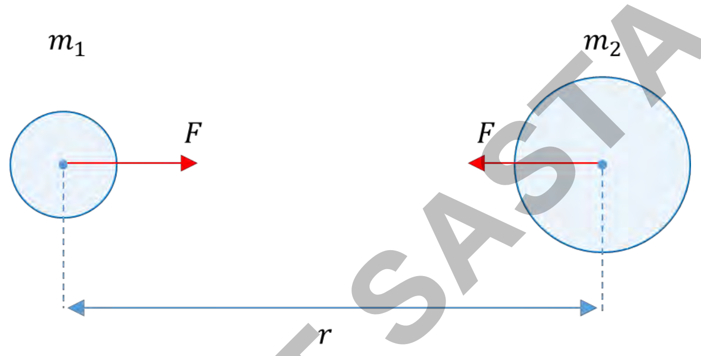
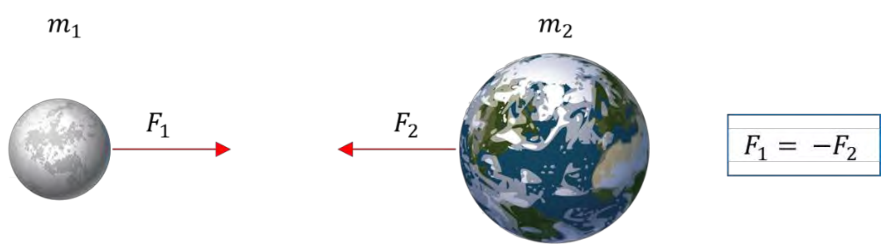


Figure 1.27: Gravitational force between two masses.

The magnitude of the gravitational force between two masses is calculated using the Universal Law of Gravitation that was first published by Isaac Newton in 1687. Newton's Universal Law of Gravitation states that the gravitational force:

- is a mutual, attractive force between two masses;
- is directly proportional to the product of the two masses ($F \propto m_1 m_2$);
- is inversely proportional to the square of the distance between the centres of the two masses ($F \propto \frac{1}{r^2}$);
- acts along a line between the centres of the two masses.

The gravitational force is consistent with Newton's Third Law of motion in that the force exerted by m_1 on m_2 is equal in magnitude but opposite in direction to the force exerted by m_2 on m_1.



The magnitude of the gravitational force between two masses is calculated using the formula for Newton's Universal Law of Gravitation.

Formula	$F = G\frac{m_1 m_2}{r^2}$

Symbol	Variable	SI unit
F	Gravitational force	(N)
m_1	Mass of object 1	(kg)
m_2	Mass of object 2	(kg)
r	Distance between the centres of the two objects	(m)
G	The gravitational constant	($N.m^2.kg^{-2}$)

The gravitational constant (G) is a constant of proportionality in Newton's Universal Law of Gravitation. The magnitude of the gravitational constant is approximately 6.67 x 10^{-11} $N.m^2.kg^{-2}$.

Example 1:

Two heavyweight boxers have masses of 97.5 kg and 101 kg respectively and are separated by a distance of 0.500 m.

Calculate the magnitude of the gravitational force between the two boxers.

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = 6.67 \times 10^{-11}\frac{101 \times 97.5}{(0.500)^2}$$

$$F = 2.63 \times 10^{-6}\ \text{N}$$

Example 2:

The average gravitational force between the moon (m = 7.35 x 10^{22} kg) and Earth (m = 5.97x 10^{24}kg) is 1.98 x 10^{20} N.

Calculate the average distance between the centres of the moon and the Earth.

$$F = G\frac{m_1 m_2}{r^2}$$

$$r = \sqrt{G\frac{m_1 m_2}{F}}$$

$$r = \sqrt{6.67 \times 10^{-11}\frac{7.35 \times 10^{22} \times 5.97 \times 10^{24}}{1.98 \times 10^{20}}}$$

$$r = 3.85 \times 10^{8}\ \text{m}$$

Objects with mass produce a gravitational field in the space that surrounds them.

An object with mass experiences a gravitational force when it is within the gravitational field of another mass.

Gravitational field strength, $\vec{g}$, is defined as the net force per unit mass at a particular point in the field. This definition is expressed quantitatively as $\vec{g} = \frac{\vec{F}}{m}$, hence it is equal in magnitude to the acceleration due to gravity.

The magnitude of the acceleration due to gravity at the surface of the Earth is 9.8 $m.s^{-2}$.

A **gravitational field** is a region of space surrounding a mass. Figure 1.29 shows the gravitational field around the Earth.

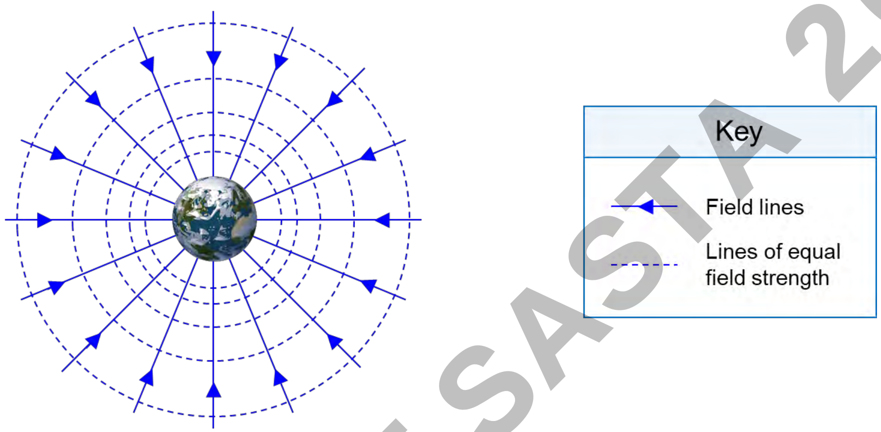


Figure 1.29: Gravitational field surrounding the Earth.

All objects with mass experience a gravitational force when they enter the gravitational field of another mass. A gravitational **field line** is a line that indicates the direction of the gravitational force that would act on a **test mass** placed in the field. A test mass is a mass that is small enough not to affect the shape of the field with its own gravitational field. The test mass accelerates in the direction of the centre of the field when placed in the gravitational field of a larger mass (M) (Figure 1.30).

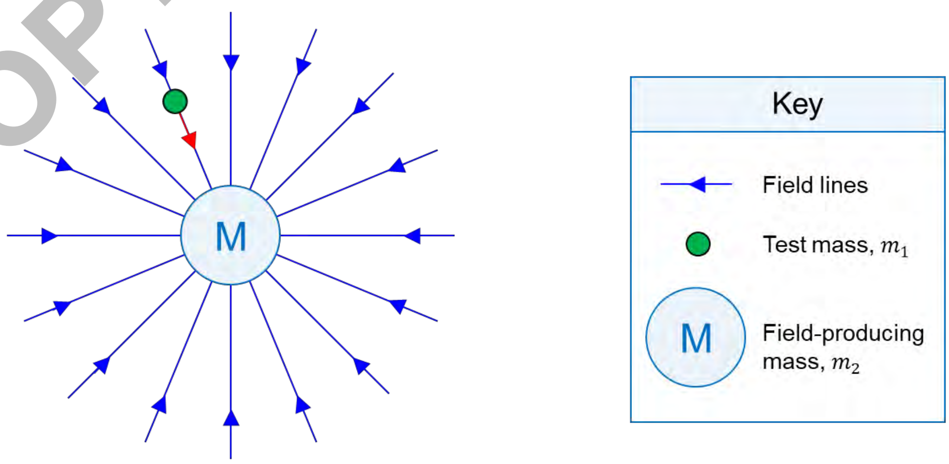
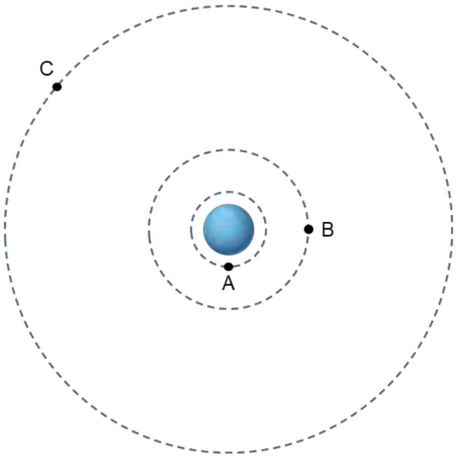


Figure 1.30: Direction of the gravitational force on a test mass placed in the field of a larger mass.

Question 34

Neptune is a planet with 14 known moons.

Some of the orbits of the known moons of Neptune are identified in the diagram below.



Letter	Name	Mass (kg)	Orbit radius (m)	Moon radius (m)
A	Despina	2.10×10^{18}	5.90×10^{7}	1.50×10^{5}
B	Proteus	5.04×10^{19}	1.18×10^{8}	4.20×10^{5}
C	Triton	2.14×10^{22}	3.55×10^{8}	1.35×10^{6}

(a) Triton is the largest moon of Neptune ($m = 1.02 \times 10^{26}$ kg).

(1) Calculate the magnitude of the gravitational force on Triton due to Neptune.

(2 marks) **KA4**

(2) State the magnitude of the gravitational force on Neptune due to Triton.

Give a reason for your answer.

____

_______________________________________ (2 marks) **KA1**

(b) Triton is composed of solid nitrogen, water, rock and metal.

Calculate the magnitude of the acceleration due to gravity near the surface of Triton.

(2 marks) **KA4**

(c) The gravitational field strength ($\vec{g}$) of Neptune decreases with distance from the centre of the planet.

(1) Define the term gravitational field strength using Neptune as an example.

____

____

_______________________________________ (1 mark) **KA1**

(2) Derive the formula used to calculate the magnitude of the gravitational field strength using Newton's Second Law of motion and the Universal Law of Gravitation.

(2 marks) **KA1**

(3) Calculate the magnitude of the gravitational field strength of Neptune at Proteus.

(2 marks) **KA4**

(4) The orbit radius of Despina is approximately half the orbit radius of Proteus.

Use proportionalities to state the gravitational field strength of Neptune at Despina.

(2 marks) **KA4**

(d) A small moon named S/2004 N1 was discovered orbiting Neptune in 2013.

The radius of the orbit of S/2004 N1 is approximately 1.05×10^{8} m

The magnitude of the gravitational force between Neptune and S/2004 N1 is 3.07×10^{15} N.

Calculate the mass of S/2004 N1.

(3 marks) **KA2**

Kepler's Laws of Planetary Motion describe the motion of planets, their moons, and other satellites.

Kepler's First Law of planetary motion: All planets move in elliptical orbits with the Sun at one focus.

Kepler's Second Law of Planetary Motion: The radius vector drawn from the Sun to a planet sweeps equal areas in equal time intervals.

Kepler's Third Law of Planetary Motion shows that the period of any satellite depends upon the radius of its orbit.

For circular orbits, Kepler's Third Law can be expressed as: $T^2 = \frac{4\pi^2}{GM}r^3$

- Describe Kepler's first two Laws of Planetary Motion.
- Use these first two Laws to describe and explain the motion of comets, planets, moons, and other satellites.
- Derive: $T^2 = \frac{4\pi^2}{GM}r^3$
- Solve problems using the mathematical form of Kepler's Third Law for circular orbits.
- Solve problems involving the use of the formulae $v = \sqrt{\frac{GM}{r}}$, $v = \frac{2\pi r}{T}$, and $T^2 = \frac{4\pi^2}{GM}r^3$.

Johannes Kepler was a German mathematician and astronomer who proposed three laws of planetary motion between 1609 and 1619. The laws of planetary motion were based on detailed observations made by Kepler on the motion of the planet Mars whilst working for the Danish astronomer Tycho Brahe (Figure 1.33).

Figure 1.33: Johannes Kepler (left). Kepler and Brahe (right).

Kepler's laws of planetary motion are three scientific laws describing the motion of planets around the Sun. However, Kepler's laws are also used to predict and explain the motion of stars, planets, moons and other satellites in areas of space outside of our own solar system.

Isaac Newton was aware of Kepler's laws of planetary motion when he derived the Universal Law of Gravitation and the Laws of Motion between 1665 and 1685.

Kepler's First Law of planetary motion

Kepler's First Law of planetary motion states that planets have elliptical orbits around the sun. An ellipse is a curve surrounding two focal points called **foci** (singular; focus) (Figure 1.34).

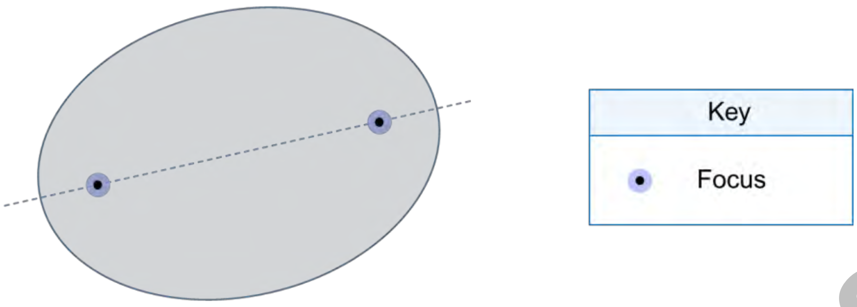


Figure 1.34: An ellipse showing foci.

The sum of the distances to the two foci (A and B) is constant for every point (P) on the curve of an ellipse (Figure 1.35).

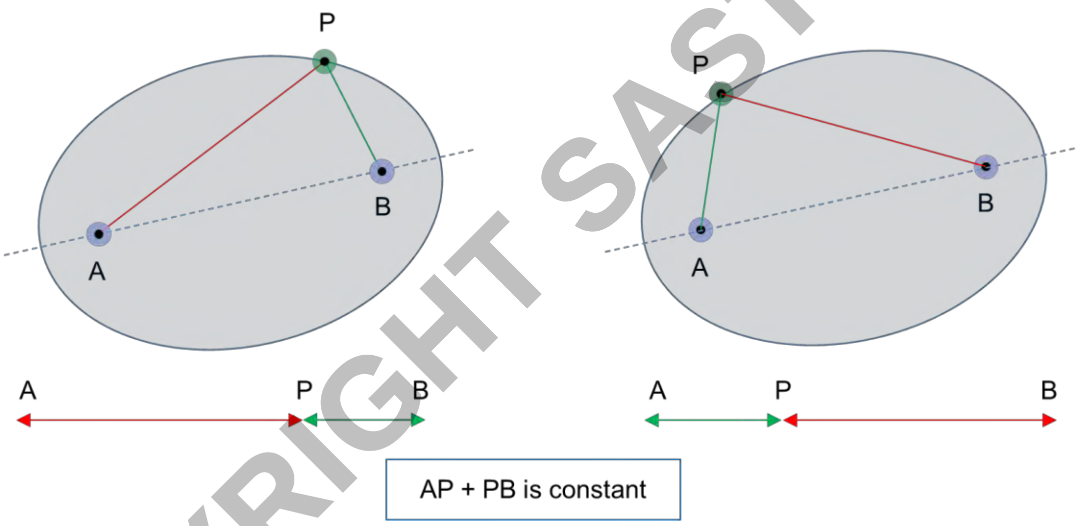


Figure 1.35: Mathematical relationship of points A, B and P on an ellipse.

Kepler's First Law of planetary motion states that the orbit of every planet is an ellipse with the Sun at one focus (Figure 1.36).

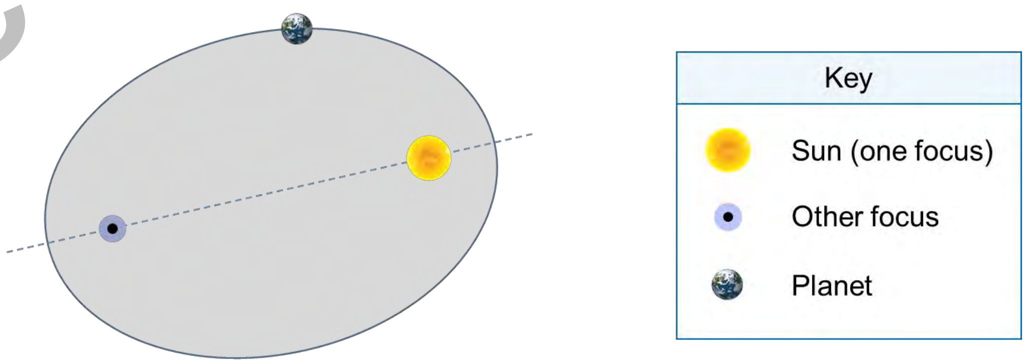


Figure 1.36: Kepler's first law of planetary motion.

Kepler's Second Law of planetary motion

Kepler's Second Law of planetary motion describes the way in which a planet moves in its elliptical orbit. The Second Law states that as the planet orbits the sun, a line joining the planet and Sun will sweep out equal areas in equal intervals of time (Figure 1.37).

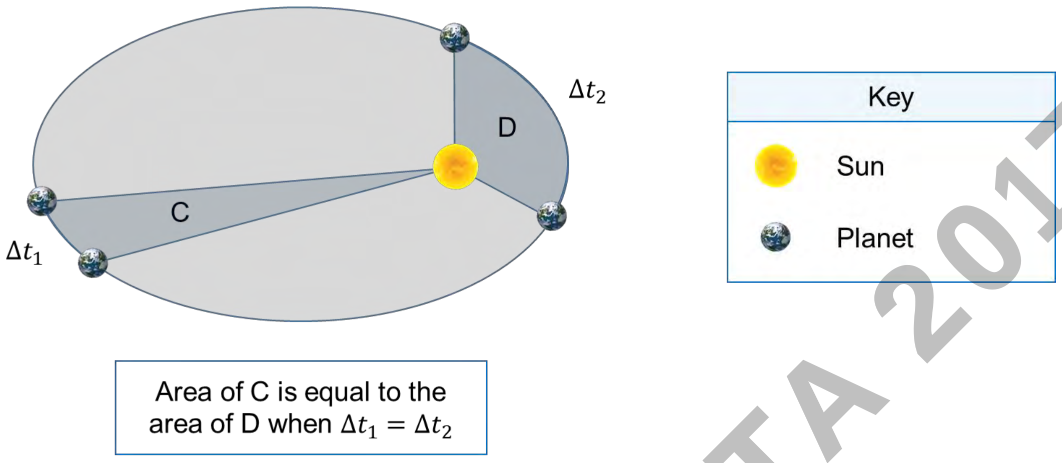


Figure 1.37: Kepler's second law of planetary motion.

The Second Law of planetary motion reveals that a planet moves a greater distance (s) in its orbit in the same time interval (Δt) when it is closer to the sun (Figure 1.38).

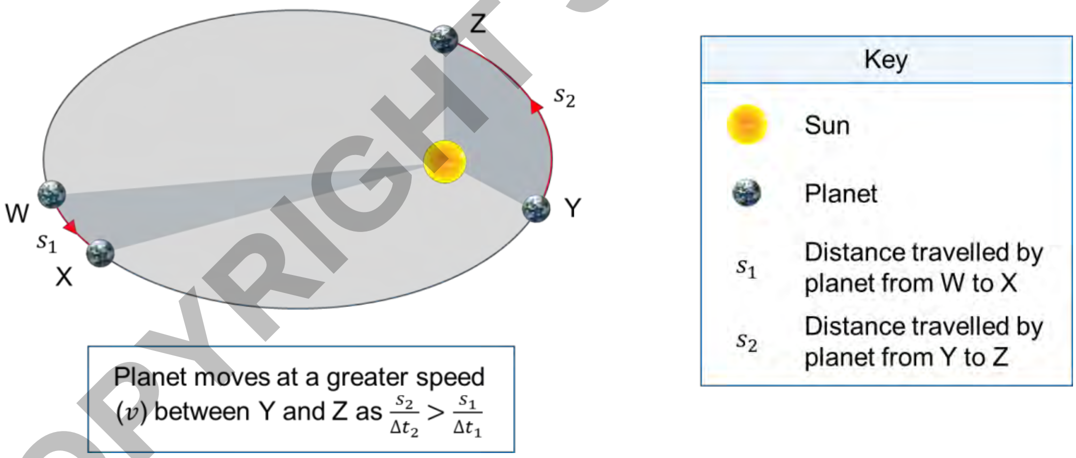


Figure 1.38: Distance travelled by a planet in its orbit in equal time intervals.

Figure 1.38 illustrates the relationship between the orbital speed (v) of a planet and its distance from the centre of the sun (r). The orbital speed decreases when the planet is farther from the sun (M) and increases when the planet is closer to the sun.

$$v^2 = \frac{GM}{r}$$

$$v^2 \propto \frac{1}{r}$$

Kepler's Third Law of planetary motion

The orbital period of a planet is the time taken to complete a full orbit. Kepler's Third Law of Planetary Motion states that the orbital period (T) of a planet depends upon the mean radius of its orbit around the sun (r). The formula showing the relationship described in Kepler's Third Law of Planetary Motion is derived below.

$$v = \frac{2\pi r}{T}$$

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$\left(\sqrt{\frac{GM}{r}}\right)^2 = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{2^2\pi^2 r^2}{T^2}$$

$$GM = \frac{4\pi^2 r^3}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

The formula shows that the square of the orbital period (T^2) is directly proportional to the cube of the radius (r^3) of the orbit of the planet ($T^2 \propto r^3$).

Example:

The Earth orbits the sun with a mean radius of 1.50×10^{11} m.

Calculate the orbital period of the Earth around the sun.

The mass of the sun is 1.99×10^{30} kg.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

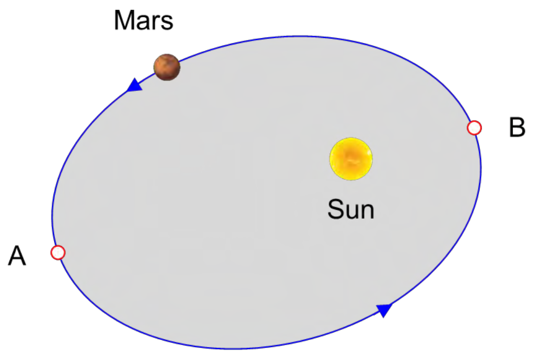
$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T = \sqrt{\frac{4\pi^2 \text{ x } (1.50 \text{ x } 10^{11})^3}{6.67 \text{ x } 10^{-11} \text{ x } 1.99 \text{ x } 10^{30}}}$$

$$T = 3.17 \text{ x } 10^7 \text{ s}$$

Question 35

The diagram below shows the orbit of Mars around the sun.



(a) State Kepler's First Law of planetary motion using Mars as the example.

____

____

______________________________________ (1 mark) **KA1**

(b) Points A and B on the diagram represents positions in the orbit where the speed of Mars is different.

(1) Derive the formula $v = \sqrt{\frac{GM}{r}}$ for the tangential speed of Mars in its orbit around the sun.

(3 marks) **KA1**

(2) Calculate the tangential speed of Mars when the distance between the centres of the sun and Mars 2.28×10^{11} m.

The mass of the sun is approximately 1.99×10^{30} kg.

(2 marks) **KA4**

(3) State and explain whether the tangential speed of mars is greater at position A or B.

____

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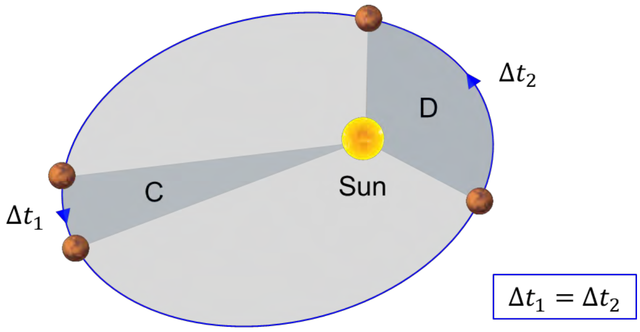
______________________________________ (2 marks) **KA1**

(c) The diagram below shows the distance travelled by Mars at two different locations in its orbit around the sun.



State Kepler's Second Law of planetary motion using the orbit of Mars as the example.

____

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____ (1 mark) **KA1**

(d) Kepler's Third Law of planetary motion describes the relationship between the orbital period (T) and the average distance between the centres of the planet and the sun (r).

(1) State Kepler's Third Law of planetary motion using the orbit of Mars as the example.

____

____ (1 mark) **KA1**

(2) Derive the formula $T^2 = \frac{4\pi^2 r^3}{GM}$ for the orbital period of Mars around the sun.

(3 marks) **KA1**

(3) Calculate the orbital period of Mars.

The average distance between the centres of the Mars and the sun is 2.28 x 10^{11} m.

Give the answer is Earth days (one earth day is 86 400 s).

(3 marks) **KA2**

Explain why a satellite in a geostationary orbit must have an orbit in the Earth's equatorial plane, with a relatively large radius and in the same direction as the Earth's rotation.

- Explain the differences between polar, geostationary, and equatorial orbits. Justify the use of each orbit for different applications.
- Perform calculations involving orbital periods, radii, altitudes above the surface, and speeds of satellites, including examples that involve the orbits of geostationary satellites.

Many artificial satellites used in communications, weather, and surveillance move in circular orbits around the Earth. A satellite in a circular orbit is moving in uniform circular motion with the gravitational force (F) causing the centripetal acceleration (Figure 1.39).

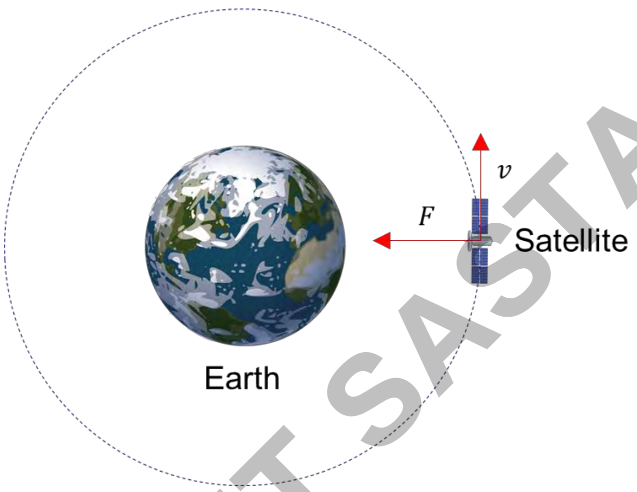


Figure 1.39: Artificial satellite in orbit around the Earth.

Only circular orbits that coincide with the centre of the Earth are possible as the gravitational force causing the centripetal acceleration of a satellite acts along a line between the centres of the Earth and satellite (Figure 1.40).

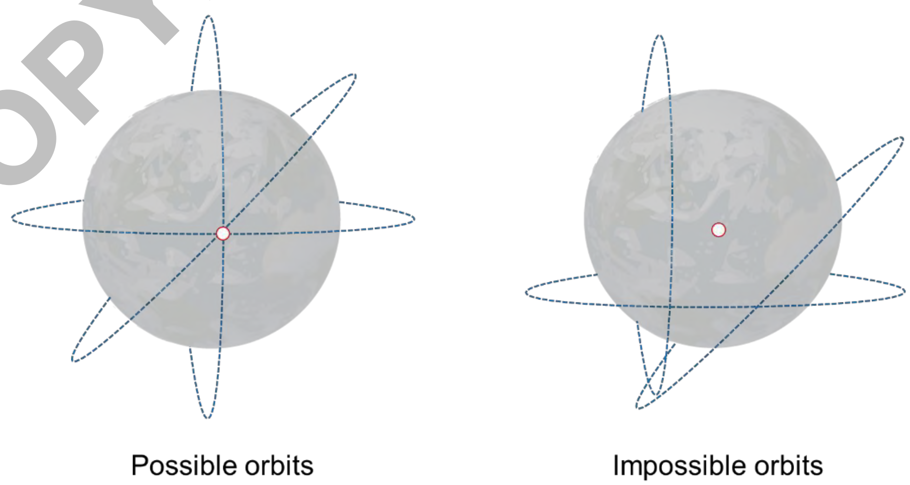


Figure 1.40: Possible (left) and impossible (right) circular orbits of Earth satellites.

Geostationary satellites

A geostationary orbit is a circular orbit at an altitude (h) of approximately 36 000 km (35,786) above the equator in the direction of Earth's rotation (west to east). An artificial satellite in a geostationary orbit is called a **geostationary satellite** (Figure 1.41).

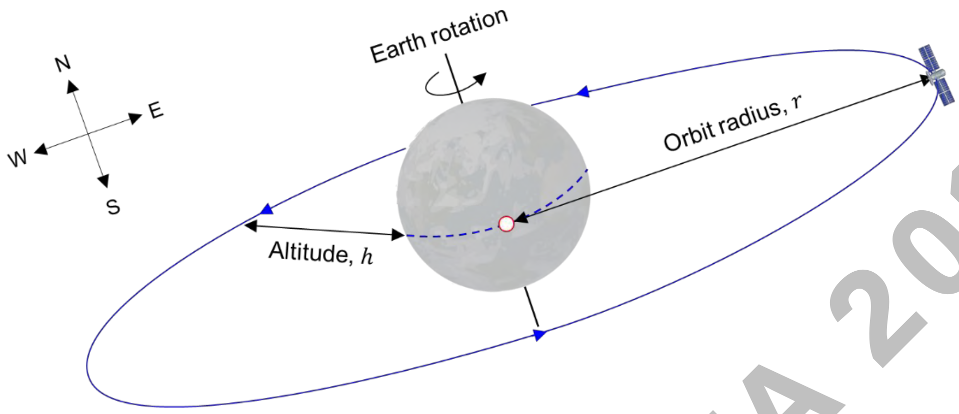


Figure 1.41: Geostationary satellite orbiting Earth.

A geostationary satellite orbits from west to east in the direction of Earth's rotation with a period of 24 hours. Since the orbital period of a geostationary satellite is equal to the Earth's rotational period (approximately 24 hours), the satellite maintains the same fixed position above the surface of the Earth. The satellite must orbit in an equatorial plane at a large altitude to maintain an orbital period that is equal to the rotational period of Earth and for the centres of the Earth and satellite to coincide such that the gravitational force causes the centripetal acceleration of the satellite. The altitude (h) of a geostationary satellite is calculated below.

Note: The period (T) of a geostationary satellite is 24 hours = 86 400 s and the radius of Earth at the equator (r_{Earth}) is 6.37 x 10^6 m.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{6.67 \text{ x } 10^{-11} \text{ x } 5.97 \text{ x } 10^{24} \text{ x } (86\,400)^2}{4\pi^2}}$$

$$r = 4.22 \text{ x } 10^7 \text{ m}$$

$$h = \mathrm{r} - \mathrm{r}_{\mathrm{Earth}}$$

$$h = 4.22 \text{ x } 10^7 - 6.37 \text{ x } 10^6$$

$$h = 3.59 \text{ x } 10^7 \text{ m} \quad (35\,900 \text{ km})$$

Advantages of geostationary satellites

Some advantages of geostationary satellites are listed in the table below.

Advantage	Explanation
Antenna does not have to be repositioned	A satellite antenna does not have to be repositioned to send and receive a transmission as the satellite is in a fixed position relative to Earth's surface.
No loss in communication due to satellite orbit	There is no loss in communication as the satellite does not move to a position that is out of sight of the antenna during its orbit.
Can monitor a large area	The large altitude of a geostationary orbit allows a geostationary satellite to monitor a large area of the Earth which is effective for tracking extreme weather events including storms and hurricanes (Figure 1.42).

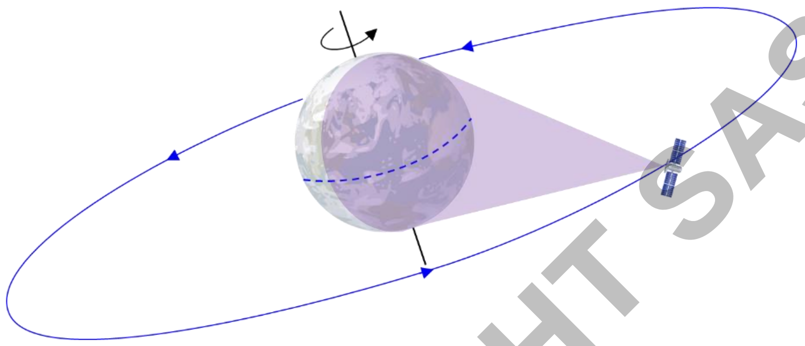


Figure 1.42: Viewing area of a geostationary satellite (left) and satellite photo (right).

Disadvantages of geostationary satellites

Some limitations of geostationary satellites are identified in the table below.

Disadvantage	Explanation
Signal loss at higher latitudes	The equatorial orbit of geostationary satellites creates difficulty in sending and receiving signals at higher latitudes due to atmospheric refraction and the satellite being out of sight due to the low angle at which the satellite views the antenna.
Time delays for communications	Geostationary satellites are far enough away from Earth that there is a time delay of approximately half a second for a signal being sent from one ground-based transmitter to the satellite and back to another ground-based transmitter.
Limited resolution	The altitude of the orbit limits the resolution of the images obtained by weather satellites that are interpreted by meteorologists.

Polar orbit satellites

A polar orbit satellite has an orbit that is above both poles of the planet being orbited on each revolution. Polar orbit satellites pass over the North and South Poles of the Earth at altitudes ranging from 300 to 5000 km (Figure 1.43).

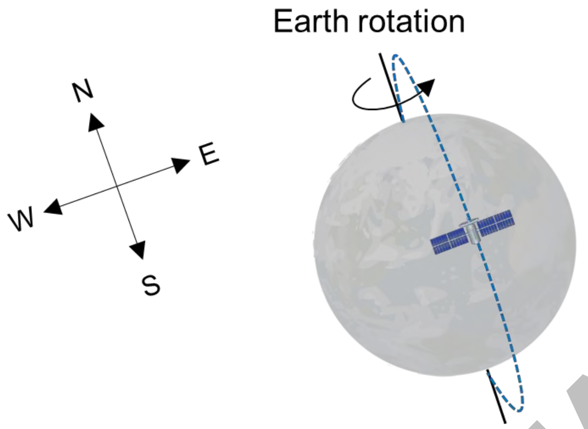


Figure 1.43: Polar orbit satellite orbiting Earth.

They orbital period of a polar orbit satellite is much shorter than the orbital period of a geostationary satellite given the lower altitude. The orbital period (T) of a satellite orbiting the poles of Earth at an altitude of 850 km is approximately 102 minutes meaning that the satellite orbits the poles approximately 14 times a day.

A polar orbit satellite surveys a region (called a swathe) of the Earth's surface that is directly beneath its orbit. Since the Earth is rotating from west to east, the orbit of the satellite is approximately 2800 km west of the previous region as it completes subsequent revolutions of the poles.

Figure 1.44 shows imaging swathes captured by a polar orbit satellite over 24 hours.

Figure 1.44: imaging swathes captured by a polar orbit satellite.

Advantages of polar orbit satellites

Some advantages of polar orbit satellites are listed in the table below.

Advantage	Explanation
Global coverage	Polar orbit satellites observe nearly every point the Earth's surface once a day which is necessary for weather prediction and climate studies.
Higher resolution images	The low altitude of a polar orbit satellite allows for higher resolution images to be captured which is useful for surveillance and monitoring weather patterns (Figure 1.45).

Figure 1.45: Photographs taken by polar orbit satellites.

Disadvantages of geostationary satellites

Some limitations of polar orbit satellites are identified in the table below.

Disadvantage	Explanation
Friction (aerodynamic drag)	The motion of a polar orbit satellite is affected by aerodynamic drag as it passes through gases in Earth's atmosphere. Aerodynamic drag causes a decrease in speed of a polar orbit satellite which effects the altitude and orbital period of the satellite over time. However, given the low density of the gases in Earth's atmosphere at high altitudes, the change in orbital speed is minimal for most polar orbits.
Inconsistent monitoring	Reconnaissance satellites are polar orbit satellites used in military and environmental surveillance. Reconnaissance satellites cannot provide continuous viewing of one location as they are in constant motion relative to the Earth.

Relative speed

It is important to understand that motion is relative, and that motion can only be described relative to an observer's frame of reference. The examples below are provided to familiarise you with the concept of relative motion.

Example 1:

A car moving uniformly at 25 m.s^{-1} drives past a truck that is stationary on the side of the road (Figure 1.48).

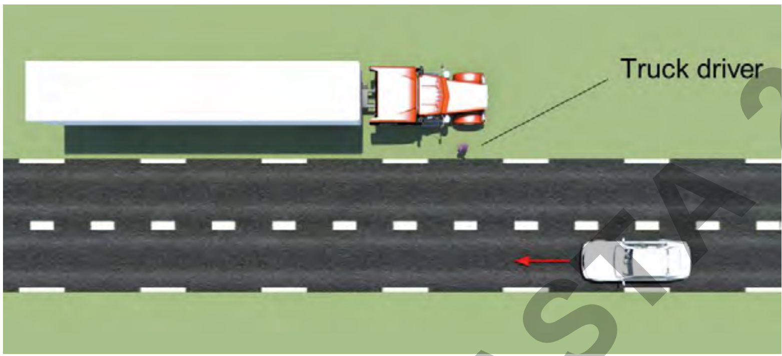


Figure 1.48: Aerial view of a car driving by a truck that is stationary on the side of the road

In the reference frame of the driver of the car:

- the car is at rest, and;
- the truck is moving towards the car at a speed of 25 m.s^{-1}.

In the reference frame of the driver of the truck:

- the truck is at rest, and;
- the car is moving towards the truck at a speed of 25 m.s^{-1}.

The reference frames of the two drivers are shown in Figure 1.49.

Reference frame of car driver

Reference frame of truck driver

Figure 1.49: Reference frames of the two drivers in Example 1.

It is more accurate to state that the truck and car are moving relatively to each other at 25 m.s^{-1} or that the relative speed of the inertial reference frames is 25 m.s^{-1}.

Example 2:

A car moving uniformly at 25 m.s^{-1} drives past a truck that is moving uniformly at 25 m.s^{-1} in the opposite direction (Figure 1.50).

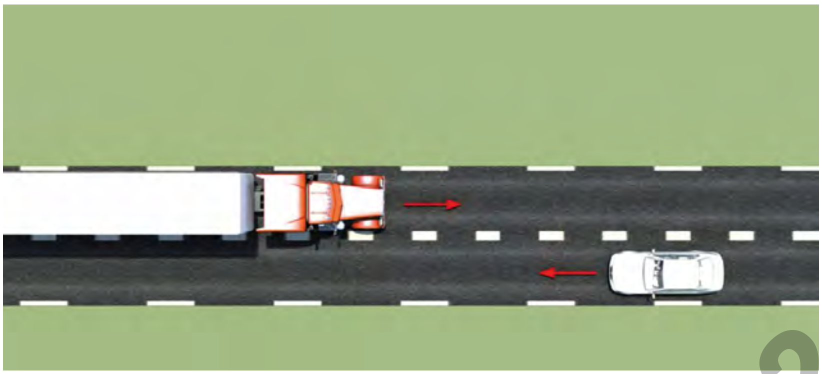


Figure 1.50: Aerial view of the car and truck driving at a constant speed in opposing directions.

In the reference frame of the driver of the car,

- the truck is moving towards the car at a speed of 50 m.s^{-1} (25 + 25).

In the reference frame of the driver of the truck,

- the car is moving towards the truck at a speed of 50 m.s^{-1} (25 + 25).

It is more accurate to state that the truck and car are moving relatively to each other at 50 m.s^{-1} or that the relative speed of the inertial reference frames is 50 m.s^{-1}.

Example 3:

A car moving uniformly at 25 m.s^{-1} drives alongside a truck that is moving uniformly at 25 m.s^{-1} in the same direction (Figure 1.51).

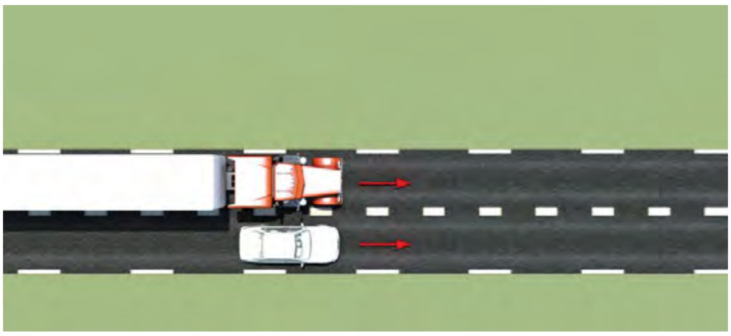


Figure 1.51: Aerial view of the car and truck in Example 3.

The truck and car appear stationary (at rest) in the reference frames of the both drivers. The relative speed of the inertial reference frames is 0 m.s^{-1}. The two reference frames are at rest with respect to each other.

Physicists assumed that electromagnetic waves moved at the speed of light relative to the ether. However, American physicists Albert Michelson and Edward Morley performed an experiment in the 1880s that disproved the existence of the ether. Furthermore, the results of the Michelson – Morley experiment showed that the speed of light in a vacuum is the same in all inertial reference frames. Physicists soon realised that the motion of electromagnetic waves seemed to be inconsistent with the Principle of Relativity. The problem is described in the examples below.

Example 1:

A person is driving along a road at 15.0 $m.s^{-1}$ when they throw a ball at a speed of 10.0 $m.s^{-1}$ in their direction of motion. An observer standing on the side of the road measures the speed of the ball as 25.0 $m.s^{-1}$ in their reference frame as the relative speed of the ball is the sum of the speeds of the car and the ball (15.0 + 10.0) (Figure 1.54).

Figure 1.54: Relative speed of the ball in the reference frame of an observer.

Example 2:

The driver is moving along a road at 15.0 $m.s^{-1}$ when they turn on a torch which emits a pulse of light in their direction of motion. The pulse of light travels away from the car in a direction parallel to their direction of motion (Figure 1.55).

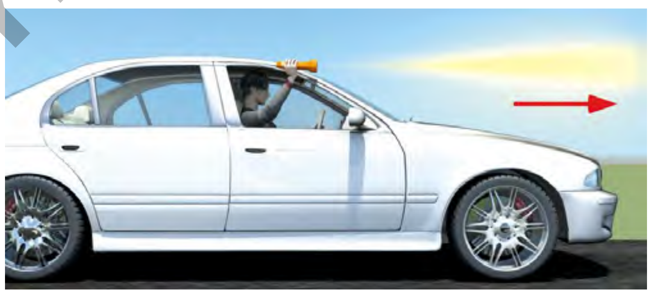


Figure 1.55: Relative speed of the light pulse in the reference frame of an observer.

According to the Principle of Relativity, the speed of light measured in the reference frame of an observer standing on the side of the road is the sum of the speeds of the car and the light pulse which is calculated as $(15.0 + 3.0 \times 10^8)$ $m.s^{-1}$. The value calculated by the observer in their reference frame is greater than the speed of light which violates the Laws of Electricity and Magnetism and disagrees with the experimental results of the Michelson – Morley experiment.

Question 39

In the future it may be possible for humans to travel to distant planets aboard a spacecraft travelling at very high speeds.

A spacecraft travelling at $0.8c$ relative to Earth carries several astronauts to a planet named Proxima Centauri b which is located approximately 4.0×10^{16} m from Earth.

(a) Show that the time taken to travel to Proxima Centauri b is approximately 5.3 years in the reference frame the Earth.

1 year = 3.154×10^{7} s

(3 marks) KA2

(b) A female astronaut is 30 years old when the spacecraft leaves Earth.

Calculate her age (in years) when she arrives at Proxima Centauri b in her reference frame aboard the spacecraft.

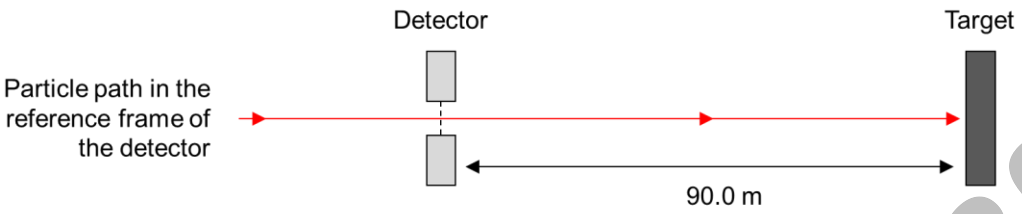
(4 marks) KA2

Question 40

A particle passes through a detector and collides with a target.

The target is 90.0 m away from the detector.

The time interval between the particle passing the detector and colliding with the target is measured as 330 ns in the reference frame of the detector.



(a) Show that the speed (v) of the particle between the detector and the target is approximately 2.73×10^8 m.s^{-1}.

(2 marks) **KA4**

(b) Calculate the time taken for the particle to travel from the detector to the target in the reference frame of the particle.

Give your answer in nanoseconds (ns).

(4 marks) **KA4**

(c) Describe motion in the reference frame of the particle.

___

___

___

___ (2 marks) **KA2**

Simultaneity

The term **simultaneous** refers to events that occur at the same time. The Special Theory of Relativity states that events that are simultaneous in one reference frame are not simultaneous in a reference frame that is moving relative to an observer.

Example:

A spacecraft is travelling uniformly at a speed of 0.70c. A crew member inside the spacecraft turns on a light that is positioned in the centre of the cabin. Observers inside the spacecraft see the light waves reach the front and rear of the cabin simultaneously (Figure 1.62).

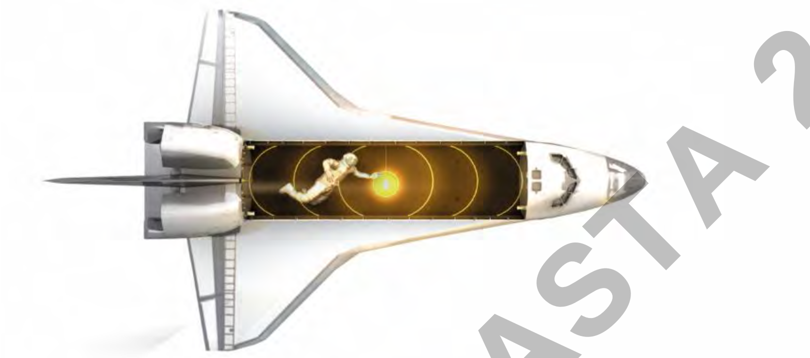


Figure 1.62: Light waves viewed by observers inside the cabin.

An astronomer on Earth is watching the spacecraft through a powerful telescope. The astronomer sees the crew member turn on the light in the centre of the cabin. In the reference frame of the astronomer on Earth, the light waves propagating towards the front of the cabin have a greater distance to travel as the front of the cabin has moved in the direction of motion of the spacecraft since the light was turned on. Conversely, the astronomer on Earth observes the light waves reaching the rear of the cabin sooner as the rear of the cabin has moved closer to the centre of the cabin since the light was turned on (Figure 1.63).

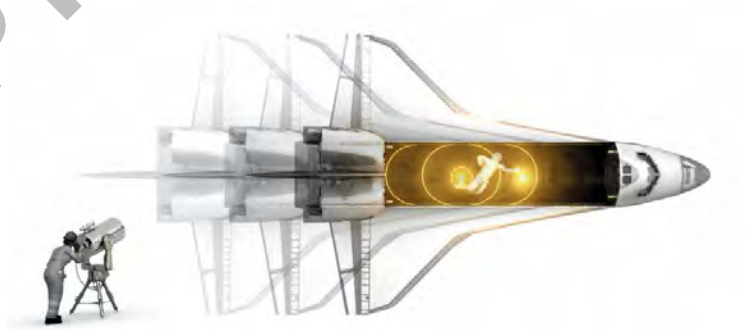


Figure 1.63: Light waves viewed by an astronomer on Earth.

To the astronomer on Earth, the light waves reach the front and rear of the cabin at different times showing that the two events are not simultaneous when the spacecraft moves relatively to the astronomer at speed 0.70c. This thought experiment shows that events that are simultaneous in one reference frame are not simultaneous in a reference frame that is moving relative to an observer.

Some subatomic particles exist in the laboratory for very short time periods before decaying. These same particles are detected as part of cosmic ray showers in the atmosphere, travelling at relativistic speeds close to the speed of light.

Time dilation effects allow these particles to travel significant distances without decay.

Given the laboratory lifetime of a subatomic particle and its relativistic speed:

- calculate time dilation factors.
- using calculations, compare the distance travelled by subatomic particles when incorporating relativistic effect.

Some subatomic particles are unstable and undergo radioactive decay after a certain time interval. The average time taken for exactly half of the unstable subatomic particles in a sample to decay is called the **half-life**. The half-life of a subatomic particle is different in the reference frame of the particle when compared to the reference frame of an observer that is moving relative to the particle due to the effect of time dilation.

Example

Muons are unstable subatomic particles created when very high energy particles called **cosmic rays** strike the upper atmosphere at an altitude of approximately 6000 m (Figure 1.64).

Figure 1.64: Production of muons in cosmic ray events in the upper atmosphere of Earth.

The half-life of a muon at rest is 2.20 μs (2.20 x 10^{-6} s) and the speed of muons produced in cosmic ray events is approximately 0.998c. The distance travelled (s) by muons travelling at 0.998c is calculated below.

$$s = vt$$
$$s = (0.998 \text{ x } 3.00 \text{ x } 10^8) \text{ x } (2.20 \text{ x } 10^{-6})$$
$$s = 659 \text{ m}$$

The muons are predicted to travel 659 m from where they are produced in the atmosphere before decaying radioactively. Therefore, it should not be possible to detect any muons at the surface of the Earth which is 6000 m from where the muons are produced in the upper atmosphere.

However, the half-life of a muon is extended in the reference frame of the Earth as the muons are moving relative to the Earth at a speed that is a significant fraction of the speed of light (c). The half-life of a muon in the reference frame of Earth is calculated below.

$$
\begin{aligned}
t &= \frac{1}{\sqrt{1 - v^2/c^2}} t_0 \\
t &= \frac{1}{\sqrt{1 - ((0.998 \text{ x } 3.00 \text{ x } 10^8)^2/(3.00 \text{ x } 10^8)^2)}} \text{ x } 2.20 \text{ x } 10^{-6} \\
t &= 15.8 \text{ x } 2.20 \text{ x } 10^{-6} \\
t &= 34.8 \text{ x } 10^{-6} \text{ s}
\end{aligned}
$$

The half-life of a muon moving at 0.998c relative to Earth is measured as 34.8 microseconds which represents which represents a 58% increase in the half-life of a muon in the reference frame of Earth.

Given the extended half-life, muons travel a greater distance (s) before decaying radioactively in the reference frame of Earth. The distance travelled by a muon moving at 0.998c relative to the Earth is calculated below.

$$
\begin{aligned}
s &= vt \\
s &= (0.998 \text{ x } 3.00 \text{ x } 10^8) \text{ x } (34.8 \text{ x } 10^{-6}) \\
s &= 1.04 \text{ x } 10^4 \text{ m } (10\,420 \text{ m})
\end{aligned}
$$

The calculation reveals that muons travel over 10 000 metres before decaying in the reference frame of Earth meaning that a greater number are detected at the surface of the Earth (Figure 1.65).

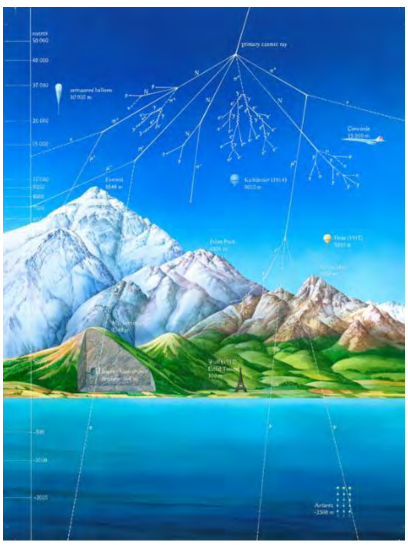


Figure 1.65: Muons detected at the surface of Earth.

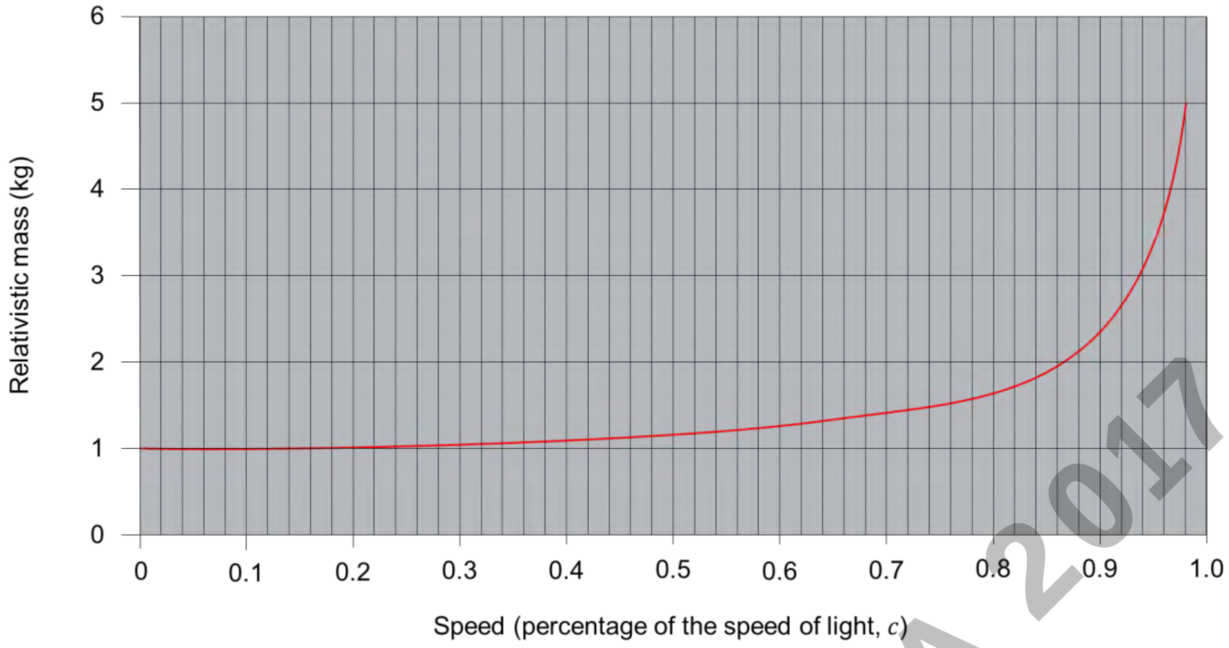


Figure 1.66: Effect of speed on the relativistic mass of a 1 kg object.

The data reveals that objects moving at a relative speed greater than $0.40c$ experience an increase in relativistic mass that is greater than 10% of the rest mass (m_0).

Nothing travels faster than light

The formula also reveals that an object cannot move at a speed greater than the speed of light in a vacuum (c). If the speed of the object was equal to the speed of light ($v = c$), the value of the denominator in the formula becomes zero and the relativistic mass becomes infinite (∞).

$$m = m_0 \text{ x } \frac{1}{\sqrt{1-(v^2/c^2)}}$$

$$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}$$

$$m = \frac{m_0}{\sqrt{1-(c^2/c^2)}}$$

$$m = \frac{m_0}{\sqrt{(1-1)}}$$

$$m = \frac{m_0}{\sqrt{0}}$$

$$m = \frac{m_0}{0}$$

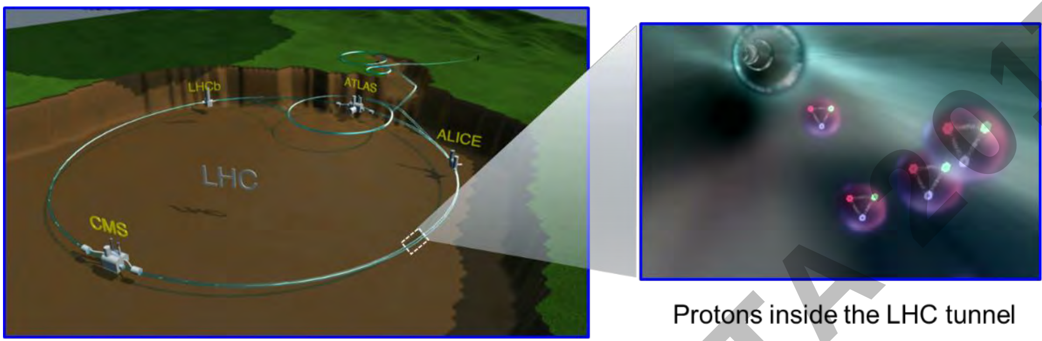
$$m = \infty$$

	Example

A proton has a rest mass (m_0) of 1.673 x 10^{-27} kg.

Protons are accelerated in the Large Hadron Collider (LHC) which is currently the world's largest and most powerful particle accelerator.

Scientists at the LHC accelerate protons to a very high speed of 0.999999991c before colliding them inside large detectors.



Protons inside the LHC tunnel

The relativistic mass of protons increases as the protons approach the speed of light.

1. Calculate the mass of the proton when it moves at 0.999999991c in the LHC.

$$m = m_0\gamma$$

$$m = m_0 \text{ x } \frac{1}{\sqrt{1-(v^2/c^2)}}$$

$$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}$$

$$m = \frac{1.673 \text{ x } 10^{-27}}{\sqrt{1-\frac{(0.999999991 \text{ x } 3.0 \text{ x } 10^8)^2}{(3.0 \text{ x } 10^8)^2}}}$$

$$m = 1.247 \text{ x } 10^{-23} \text{ kg}$$

2. Calculate the momentum of a proton travelling at 0.999999991c.

$$p = mv$$

$$p = 1.247 \text{ x } 10^{-23} \text{ x } (0.999999991 \text{ x } 3.0 \text{ x } 10^8)$$

$$p = 3.74 \text{ x } 10^{-15} \text{ kg.m.s}^{-1}$$

Question 46

Electrons travel at a speed of 0.995c in the Stanford linear accelerator (SLAC).

The rest mass of an electron is 9.11×10^{-31} kg.

(a) Show that the relativistic mass of an electron moving at 0.995c is 9.12×10^{-30} kg.

(2 marks) **KA4**

(b) Calculate the momentum of electrons moving at 0.995c.

(2 marks) **KA4**

(c) Protons are also accelerated to high speeds inside the SLAC.

The rest mass of a proton is 1.673×10^{-27} kg.

The formula used to determine relativistic mass has been partially rearranged for you.

$$\frac{v^2}{c^2} = 1 - \left(\frac{m_0}{m}\right)^2$$

Use the rearrangement to determine the speed of the protons (v) with a relativistic mass of 1.675×10^{-26} kg.

Give your answer as a percentage of the speed of light in a vacuum (c).

(4 marks) **KA4**

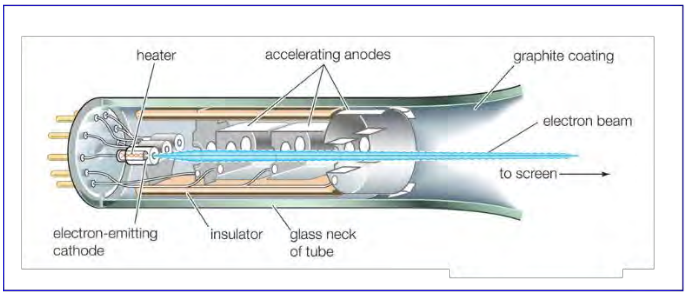
(d) Explain why it is not possible to accelerate protons and electrons to the speed of light.

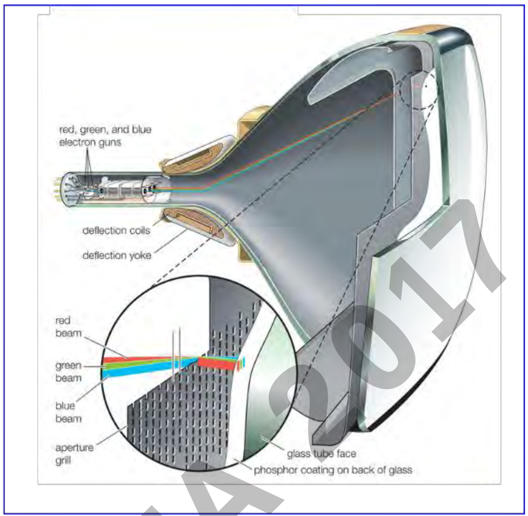
____

____ (2 marks) **KA2**

Question 47

The electrons in a cathode ray tube (CRT) television travel from an electron gun to the screen at $0.30c$.





(a) The electrons travel a distance of 0.30 m from the electron gun to the screen in the reference frame of the television.

(1) Calculate the time taken for electrons to travel from the electron gun to the screen in the reference frame of the television.

(2 marks) **KA2**

(2) Calculate the time taken for electrons to travel from the electron gun to the screen in the reference frame of the electrons.

(3 marks) **KA2**

(b) Show that the distance travelled by electrons is 0.29 m in the reference frame of the electrons.

(3 marks) **KA2**

(c) The rest mass of an electron is 9.11 x 10^{-31} kg.

(1) Show that the relativistic mass of an electron inside the CRT television is 9.55 x 10^{-31} kg.

(2 marks) **KA4**

(2) Calculate the momentum of an electron inside the CRT television.

(2 marks) **KA4**

(d) The deflection coils and deflection yoke produce a magnetic field that deflects the path of electrons in the direction of the screen.

The electron beam must be deflected to the correct position on the phosphor coated screen for the image to be formed correctly.

The angle of deflection (θ) required to focus the electron beam is given by the formula below.

$$\theta = \arctan \frac{v_1}{v_2}$$

The speed v_1 is calculated using the formula below.

$$v_1 = \sqrt{\frac{2qV}{m}}$$

Where m is the mass of an electron.

Use the information above to explain why engineers had to consider the effects of special relativity when designing CRT televisions.

____

____

____

____

____

____ (4 marks) **KA2**

Review Test 1

Question 1

The Global Positioning System (GPS) is a navigation system that provides information on the position (location) and time of an object on Earth.

GPS uses satellites that orbit the Earth in uniform circular motion at an altitude of 20.2×10^6 m.

(a) State what is meant by uniform circular motion using a GPS satellite as an example.

____

____ (1 mark) **KA1**

(b) Identify the force causing the centripetal acceleration of a GPS satellite as it orbits Earth.

____ (1 mark) **KA1**

(c) Use the diagram below to show that the acceleration of the GPS satellite is directed towards the centre of the circular path.

(3 marks) **KA4**

(d) Derive the formula $v = \sqrt{\frac{GM}{r}}$ for the tangential speed of a GPS satellite.

(2 marks) **KA1**

(e) The radius of Earth is approximately 6.37×10^6 m and the mass of Earth is 5.97×10^{24} kg.

Show that the tangential speed of the satellite is approximately 3.87×10^3 m.s^{-1}.

(3 marks) **KA4**

(f) GPS satellites orbit Earth with a fixed period (T).

(1) Derive the formula $T^2 = \frac{4\pi^2 r^3}{GM}$ for the orbital period of a GPS satellite.

(3 marks) **KA1**

(2) State Kepler's Third Law using the example of a GPS satellite orbiting Earth.

(1 mark) **KA1**

(3) Show that a GPS satellite orbits the Earth twice a day.

One Earth day is 86 400 s.

(3 marks) **KA2**

(g) GPS satellites are owned by the US Government and are operated by the US Air Force.

A GPS satellite has an onboard clock that can be controlled remotely.

The Earth is moving relatively to the GPS satellite at 3.87×10^3 m.s^{-1}.

Explain, using the Special Theory of Relativity why the US Air Force must correct the clocks aboard GPS satellites every day.

(3 marks) **KA1**

Question 2

Muons are unstable subatomic particles created when very high energy particles called **cosmic rays** strike the upper atmosphere of Earth at an altitude of approximately 6000 m.

Muons move uniformly at a speed of 0.998c relative to Earth after being produced in the upper atmosphere.

The half-life of a muon at rest is 2.20 μs

(a) Show that the maximum distance travelled by muons in the reference frame of Earth is 660 m.

(2 marks) **KA4**

(b) Show that the half-life of a muon is extended to 34.8 μs in the reference frame of Earth.

(3 marks) **KA2**

(c) Scientists set up muon detectors on the surface of the Earth.

Show a calculation and explain why scientists detect muons at the surface given that the particles travel only 660 m before decaying radioactively.

Space for calculation

Space for explanation

(4 marks) **KA2**

Question 3

An investigation was conducted to determine the effect of launch angle on the horizontal range of a projectile.

The relationship between launch angle (θ) and horizontal range (s_H) is calculated using the formula below.

$$s_H = \frac{v_0^2 \sin 2\theta}{g}$$

A projectile launcher was used to launch a steel ball at an initial velocity (v_0) that was constant between trials.

The launch angle was varied by the experimenter and the horizontal range was measured using a laser distance measurer.

The experimental results are shown below.

Launch angle, θ (°)	10	20	30	40	45
$\sin 2\theta$	0.342	0.643	0.866		1.00
Horizontal range, s_H (m)	0.872	1.64	2.21	2.51	2.55

(a) Complete the table by calculating the missing value for $\sin 2\theta$. (1 mark) **IAE2**

(b) Draw a graph with range on the vertical axis and $\sin 2\theta$ on the horizontal axis.

Include a line of best fit. (4 marks) **IAE2**

(c) Use your graph to determine the horizontal range when the launch angle was 25°.

Show a ruled interpolation and a calculation to achieve full marks for this question.

(3 marks) **IAE3**

(d) Show that the gradient of the line of best fit is approximately 2.55.

Determine the units.

(3 marks) **IAE3**

(e) State the equation of the line of best fit.

____ (1 mark) **IAE3**

(f) The acceleration due to gravity is $9.8\ m.s^{-2}$.

Use the gradient of the line of best fit and the formula on the previous page to determine the initial velocity (v_0).

(3 marks) **IAE3**

(g) The projectile underwent uni-level projection in this investigation.

State another launch angle that would give a range of 2.21 m.

____ (1 mark) **IAE3**

Question 4

Extended response question

In answering this question, you should

- communicate your knowledge clearly and concisely;
- use physics terms correctly;
- present information in an organised and logical sequence;
- include only information that is related to the question.

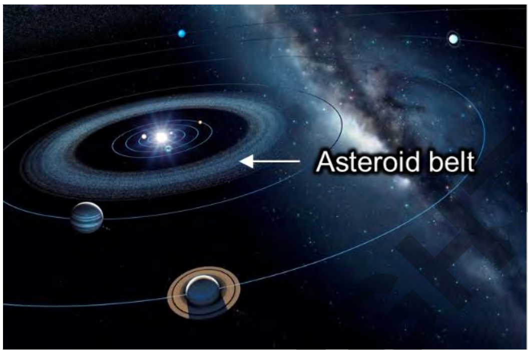
Asteroids are minor planets with diameters less than 100 kilometres.

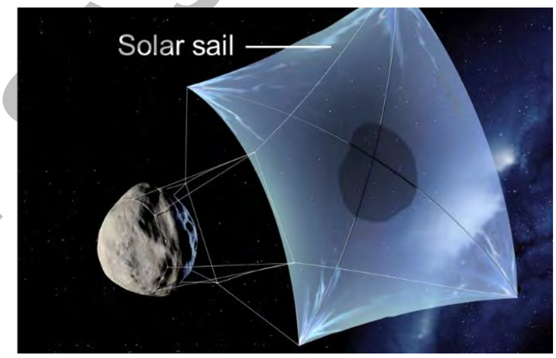
The asteroid belt is a system of over a million asteroids located between the planets Mars and Jupiter. The asteroids differ in mass and their distance from the sun.

The asteroids in the belt have elliptical orbits around the sun.

It is possible that asteroids can move out of the asteroid belt and threaten Earth.

Scientists have proposed using solar sails to deflect an asteroid from its pathway towards Earth.





- Describe the properties of the gravitational force acting on the different asteroids in the asteroid belt as they orbit the sun.
- Use the law of conservation of momentum to describe and explain how the reflection of particles of light (photons) from a solar sail can be used to deflect an asteroid.

2.1: Electric Fields

Electrostatically charged objects exert forces upon one another; the magnitude of these forces can be calculated using Coulomb's Law.

- Solve problems involving the use of $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$
- Using proportionality, discuss changes in the magnitude of the force on each of the charges as a result of a change in one or both of the charges and/or a change in the distance between them.
- Explain that the electric forces are consistent with Newton's Third Law.

The **electrostatic force** (F) is a force of attraction or repulsion between any two charges $(q_1$ and $q_2)$ that are separated by a distance (r).

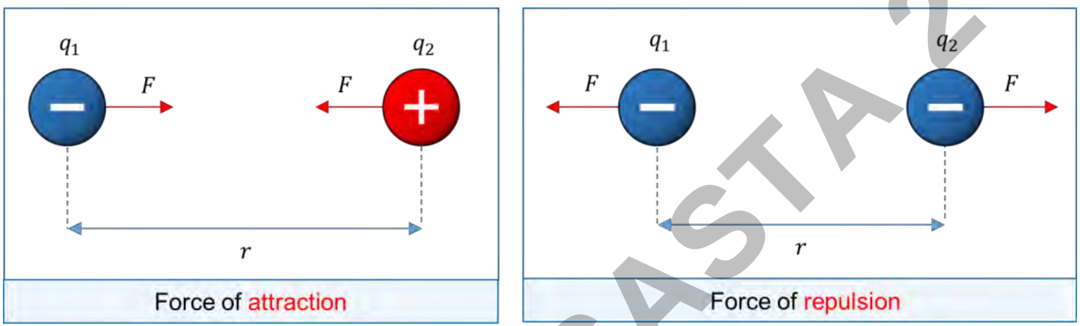


Figure 2.01: Electrostatic force between two charges.

The magnitude of the electrostatic force between two masses is calculated using **Coulomb's Law** which was first published in 1784 by French physicist Charles-Augustin de Coulomb. Coulomb's Law states that the electrostatic force:

- is **attractive** between opposite charges (e.g. a positive charge and a negative charge);
- is **repulsive** between like charges (e.g. two positive charges or two negative charges);
- is directly proportional to the product of the two charges $(F \propto q_1 q_2)$;
- is inversely proportional to the square of the distance between the two charges $(F \propto \frac{1}{r^2})$;

The electrostatic force is consistent with Newton's Third Law of motion in that the force exerted by q_1 on q_2 is equal in magnitude but opposite in direction to the force exerted by q_2 on q_1 (Figure 2.02).

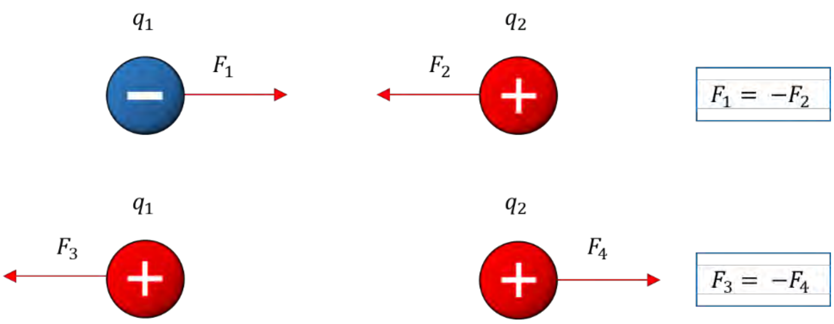


Figure 2.02: Mutual forces of attraction and repulsion between charges

The magnitude of the electrostatic force between two charges is calculated using the formula for Coulomb's Law. The SI unit of charge is the coulomb (C).

Formula	$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

Symbol	Variable	SI unit
F	Electrostatic force	(N)
q_1	Charge of object 1	(C)
q_2	Charge of object 2	(C)
r	Distance between the centres of the two objects	(m)
$\frac{1}{4\pi\varepsilon_0}$	Coulomb's constant	($N.m^2.C^{-2}$)

Coulomb's constant ($\frac{1}{4\pi\varepsilon_0}$) is a constant of proportionality in the formula for Coulomb's Law. The magnitude of Coulomb's constant is approximately 9.00 x 10^9 $N.m^2.C^{-2}$. The symbol ε_0 represents the permittivity of a vacuum which is a factor that affects the magnitude and propagation of the electric field between two charges.

Example:

Two positive charges (4.0 μC) are 1.5 cm apart in a vacuum.

1. Calculate the magnitude of the electrostatic force between the charges

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$
$$F = 9.00 \times 10^9 \frac{4.0 \times 10^{-6} \times 4.0 \times 10^{-6}}{(0.015)^2}$$
$$F = 640\ \text{N}$$

2. The charges repel and are now 4.5 cm apart.

   Use proportionalities to calculate the magnitude of the electrostatic force.

Notes

- The distance between charges has been increased by a factor of 3 times $\left(\frac{4.5\ \text{cm}}{1.5\ \text{cm}}\right)$ which means that the force is reduced by a factor of $\left(\frac{1}{r^2}\right)$ which is $\left(\frac{1}{3^2}\right)$ or $\left(\frac{1}{9}\right)$.

$$F \propto \frac{1}{r^2}$$
$$F = 640 \times \frac{1}{9}$$
$$F = 71\ \text{N}$$

Isolated point charges

A point charge is an electric charge that is concentrated in a single point in space. Point charges have either a positive or negative sign of charge. Isolated point charges have **radial fields** with the field lines drawn from the central point charge and extending outwards like the radii of a circle, as shown in Figure 2.05.

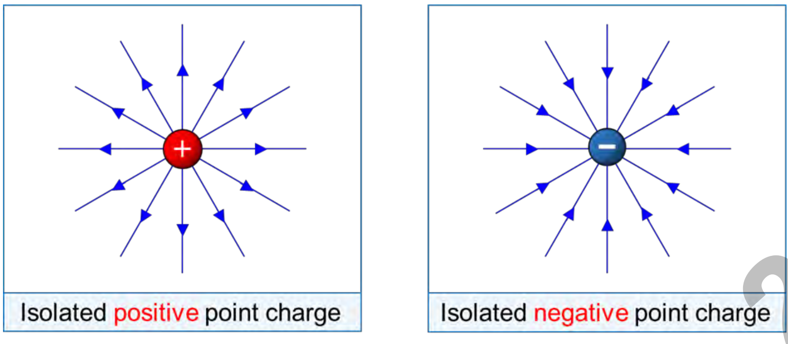


Figure 2.05: The electric field around isolated point charges

The strength of the electric field around a charge is represented by the number of field lines per unit area. Stronger fields are represented by drawing a greater number of field lines per unit area as shown in Figure 2.06.

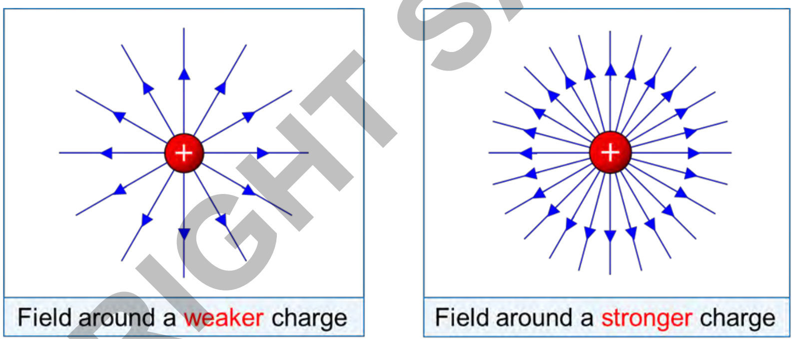


Figure 2.06: Number of field lines per unit area around isolated point charges.

Electric dipole

An electric dipole consists of two point charges that are separated by a short distance. Each point charge in an electric dipole is affected by the electric field of the other. Figure 2.07 shows the electric field surrounding dipoles composed of like charges.

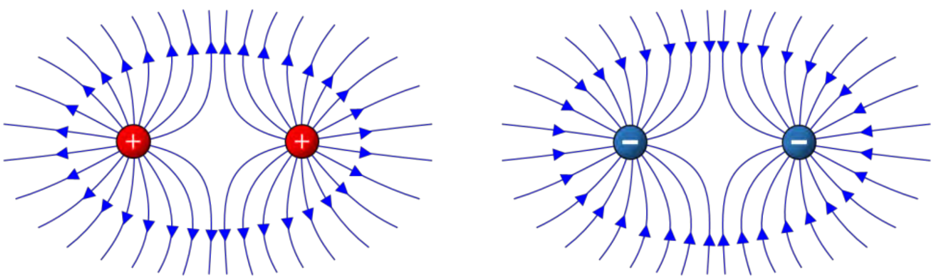


Figure 2.07: Electric fields surrounding dipoles composed of like charges

Figure 2.07 shows that the electric field lines diverge in the area between two point charges. This represents the electrostatic force of repulsion between like charges.

Figure 2.08 shows the electric field surrounding a dipole composed of opposite charges. Note that field lines are directed towards the negative charge and away from the positive charge. Also note that this a two-dimensional sketch of a three-dimensional electric field.

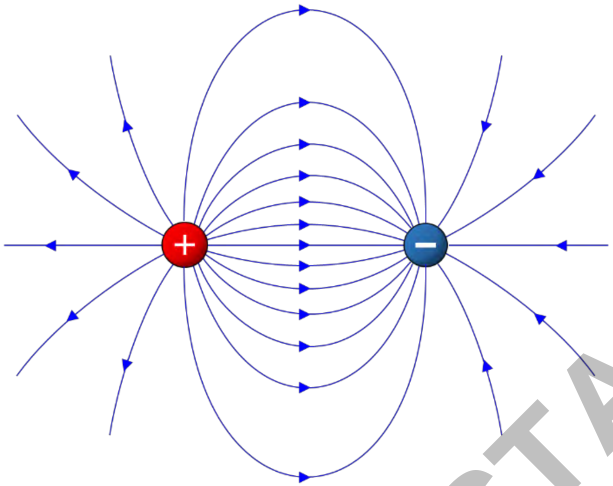


Figure 2.08: Electric field surrounding dipoles composed of like charges

Oppositely charged parallel plates

Oppositely charged parallel plates are an example of a charged object rather than a point charge. The electric field is uniform between the two charged plates and this is represented by field lines that are evenly spaced (Figure 2.09).

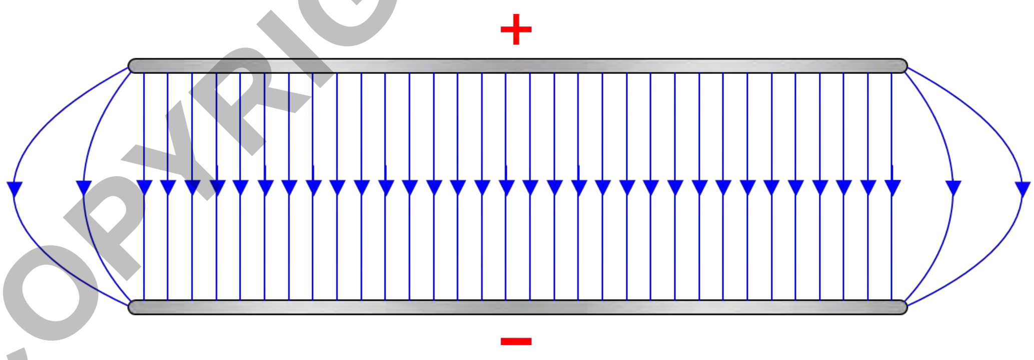


Figure 2.09: Electric field between oppositely charged parallel plates

Note that the electric field is not uniform at the ends of the plates as the plates are finite in length. The field lines are not evenly spaced and are further apart at the ends of the plates indicating that the electric field is weaker in these regions.

The motion of electric charges in the electric field between two charged plates is explored in Topic 2.2: *The Motion of Charged Particles in Electric Fields.*

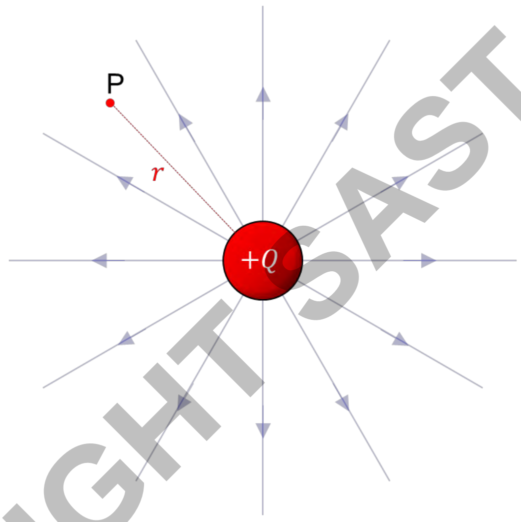
Electric field strength

The electric field strength ($\vec{E}$) at any point in the field is a measure of the electrostatic force (F) per unit test charge (q) and is calculated using the formula below. The SI unit of electric field strength is newtons per coulomb ($N.C^{-1}$).

$$\vec{E} = \frac{\vec{F}}{q}$$

The electric field strength at some position in the electric field surrounding a point charge is calculated using an alternate expression which is derived below.

Consider measuring the electric field strength at position P that is a distance (r) from an isolated point charge (Q).



A test charge (q) is placed at position P in the field to determine the electric field strength (E) at this location. The electrostatic force (F) on the test charge (q) at position P in the field around the central charge (Q) can be calculated using two formulae that are mathematically equivalent.

$$F = Eq \qquad F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

A formula for calculating electric field strength is derived using these two formulae.

$$Eq = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$$

$$E\cancel{q} = \frac{1}{4\pi\varepsilon_0}\frac{\cancel{q}Q}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Question 58

Aeroplanes are struck by lightning during flight approximately once every 1000 hours.

The outer casing of an aeroplane is made from metal such as aluminium.

(a) Explain why passengers inside the cabin of the plane are unharmed when lightning strikes.

(2 marks) **KA2**

(b) Explain why lightning usually strikes at the wing tips of the aeroplane as shown in the diagram above.

(2 marks) **KA2**

(c) Electric current passes through the outer casing and exits from the stabilisers and rudder at the tail end of the plane.

The electric field at the tail of the plane causes corona discharge in the surrounding air.

Describe the phenomenon of corona discharge in air.

(2 marks) **KA2**

Electric potential difference (ΔV) is defined as the work done per unit charge $\left(\frac{W}{q}\right)$ in moving the test charge between points A and B in the field. The units of electric potential difference are joules per coulomb ($J.C^{-1}$) or volts (V).

Formula	$\Delta V = \frac{W}{q}$

Symbol	Variable	SI unit
ΔV	Electric potential difference between two points in the field	(V)
W	Work done on the test charge	(J)
q	Charge of the test charge	(C)

Example:

Calculate the work done in moving an electron across a potential difference of 100 kV in an X-ray tube.

W	=	$q\Delta V$
W	=	$1.6 \times 10^{-19} \times 100 \times 10^{3}$
W	=	1.6×10^{-14} J

The electronvolt (eV)

The electronvolt (symbol eV) is a unit of energy equal to approximately 1.6×10^{-19} joules. One electronvolt (1 eV) is the change in energy that occurs when an electron is moved across an electric potential difference of one volt.

It is common for the energy of a charged particle such as an electron or proton to be expressed in units of electronvolts. The units of energy are interconverted between joules and electronvolts using the formulae below.

From energy in joules (E_J) to energy in electronvolts ($E_{(eV)}$):

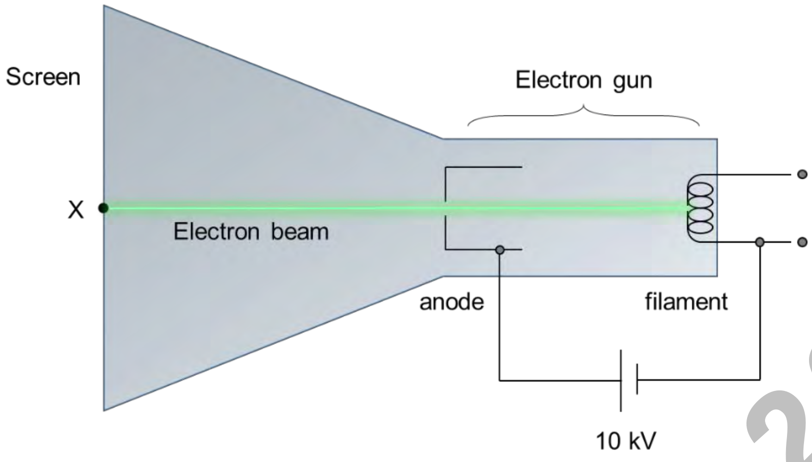
$$E_{(eV)} = \frac{E_J}{1.6 \times 10^{-19}}$$

From energy in electronvolts ($E_{(eV)}$) to energy in joules ($E_{(J)}$):

$$E_{(J)} = E_{(eV)} \times 1.6 \times 10^{-19}$$

Question 59

The diagram is of a simplified cathode ray tube (CRT).



Electrons are accelerated from the filament to the anode through potential difference of 10 kV.

A beam of electrons emerges from a hole in the anode and strikes the screen at point X.

There is no electric field between the anode and the screen.

(a) The electrons are initially at rest inside the filament.

Calculate the change in kinetic energy of the electrons from the filament to the anode.

Give your answer in joules (J) and electronvolts (eV).

(3 marks) **KA4**

(b) Calculate the maximum speed of the electrons emerging from the hole in the anode.

(3 marks) **KA4**

(c) Explain why the speed of the electrons is constant between the anode and point X on the screen.

____

____

____ (2 marks) **KA2**

The magnitude of the electric field strength (away from the edges) between two oppositely charged parallel plates a distance d apart, where ΔV is the potential difference between the plates, is given by the formula: $E = \frac{\Delta V}{d}$.

The force on a charged particle moving in a uniform electric field is constant in magnitude and direction, thus producing a constant acceleration.

- Solve problems using $E = \frac{\Delta V}{d}$
- Derive the formula $\vec{a} = \frac{q\vec{E}}{m}$
- Solve problems using $\vec{a} = \frac{q\vec{E}}{m}$ and the motion formulae for the movement of charged particles parallel or antiparallel to a uniform electric field.
- Describe the motion of charged particles parallel or antiparallel to a uniform electric field.

Figure 2.22 shows the electric field in the region between two oppositely charged parallel plates that are a fixed distance (d) apart.

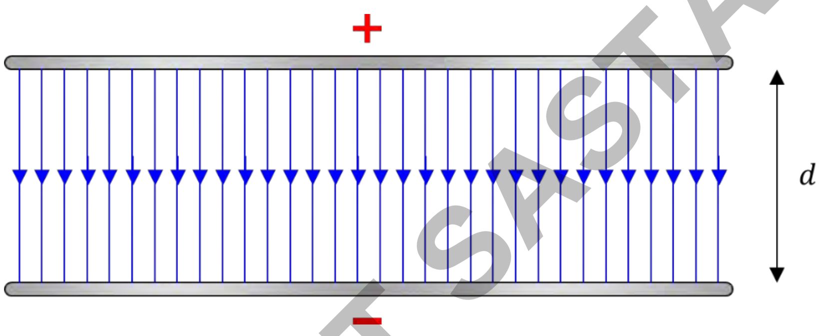


Figure 2.22: Electric field between oppositely charged parallel plates

The electric field between the plates is uniform which means that the electric field strength (E) is constant in magnitude and direction. The work done (W) in moving a test charge (q) between the plates is calculated using either of the two formulae below.

$$W = \Delta V q \qquad\qquad W = Fd$$

The two formulae are mathematically equivalent and can be used to derive a formula to calculate the magnitude of the electric field strength between the plates.

$$\begin{aligned} \Delta V q &= Fd \\ \Delta V \cancel{q} &= E\cancel{q}d \\ \Delta V &= Ed \\ E &= \frac{\Delta V}{d} \end{aligned}$$

The units of measurement for the electric field strength between the plates are newtons per coulomb ($N.C^{-1}$) or volts per metre ($V.m^{-1}$).

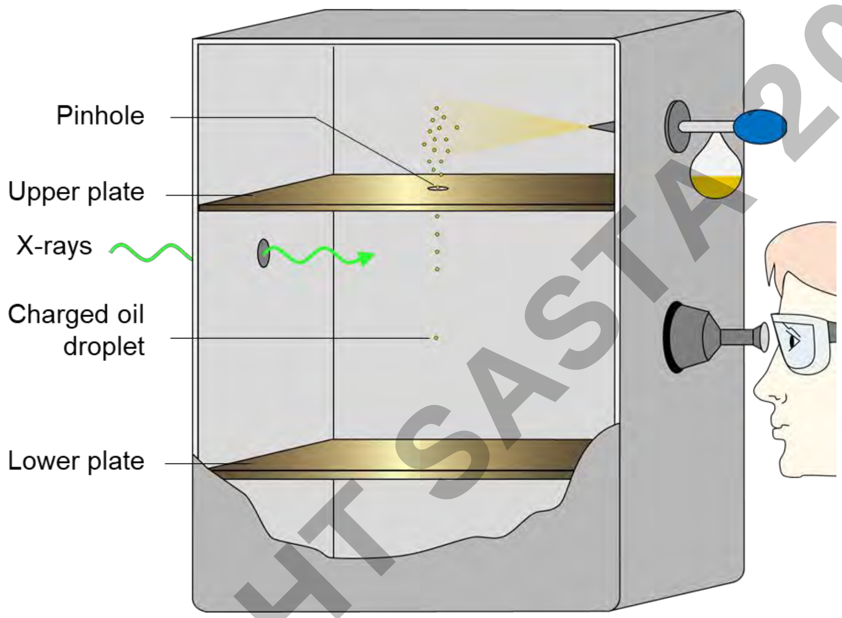
Question 60

In 1910, American physicist Robert Millikan performed an experiment that accurately determined the charge on an electron.

The experiment involved spraying neutral oil droplets into a uniform electric field between two parallel plates.

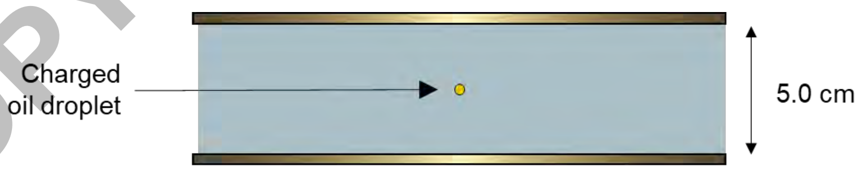
The oil droplets fell vertically under gravity through a pinhole in the upper plate.

The oil droplets were ionised using a beam of X-rays as they fell from the upper to the lower plate.



Millikan observed some oil droplets at rest between the upper and lower plate.

The diagram below shows a positively charged oil droplet at rest between two parallel plates.



(a) The electric potential difference between the plates is 2.5 kV.

The electric potential of the lower plate is 0 V.

State the charge (+ or −) and electric potential (V) of the upper plate.

____

____ (2 marks) **KA2**

(b) Show that the magnitude of the electric field strength between the plates is 5.0×10^4 V.m^{-1}.

(2 marks) **KA4**

(c) State why the electric field strength is 5.0×10^4 V.m^{-1} at all positions between the two plates.

____

____ (1 mark) **KA1**

(d) Describe and explain the motion of oil droplets that were not ionised by X-rays in the region between the upper and lower plate.

____

____

____

____

____ (2 marks) **KA2**

(e) The mass of one oil droplet at rest between the two plates was 1.63×10^{-15} kg.

(1) Calculate the charge (q) on the oil droplet.

The acceleration due to gravity is 9.8 m.s^{-2}.

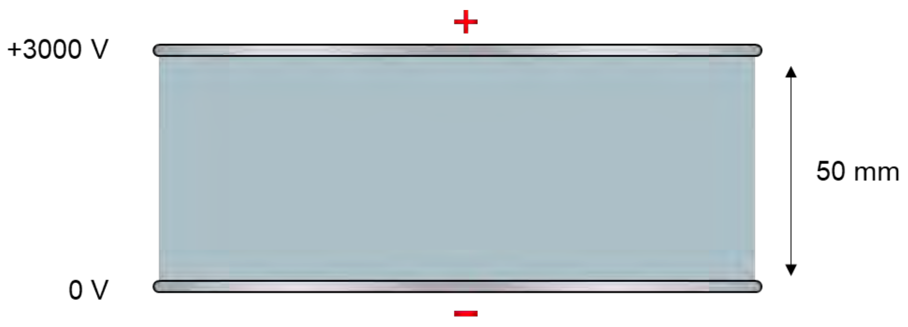
(3 marks) **KA4**

(2) Calculate the number of electrons that were ionised from the oil droplet at rest between the two plates.

(1 mark) **KA4**

Question 61

The diagram below shows two parallel plates that are 50 mm apart in a vacuum.



(a) Sketch the electric field in the region between the two plates. (2 marks) **KA1**

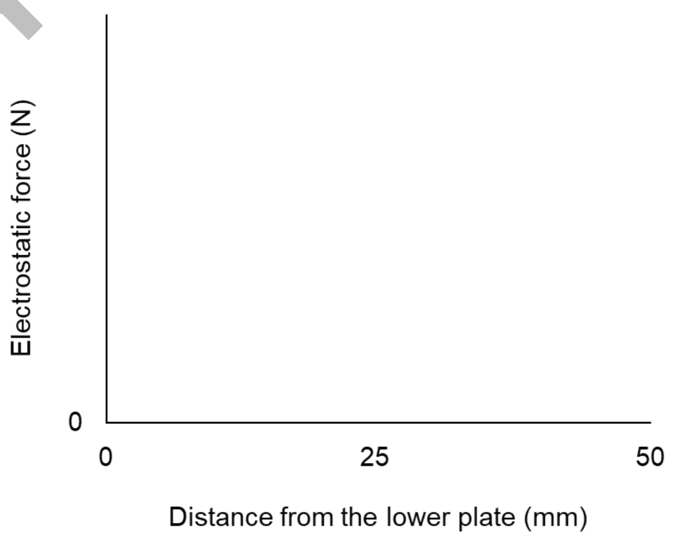
(b) A beam of electrons moves in the region between the two plates.

The potential difference between the plates is 3000 V.

Show that the electrostatic force on an electron in the beam is 9.6×10^{-15} N.

(4 marks) **KA4**

(c) Complete the graph below by showing how the electrostatic force on an electron varies with the distance of from the lower plate.



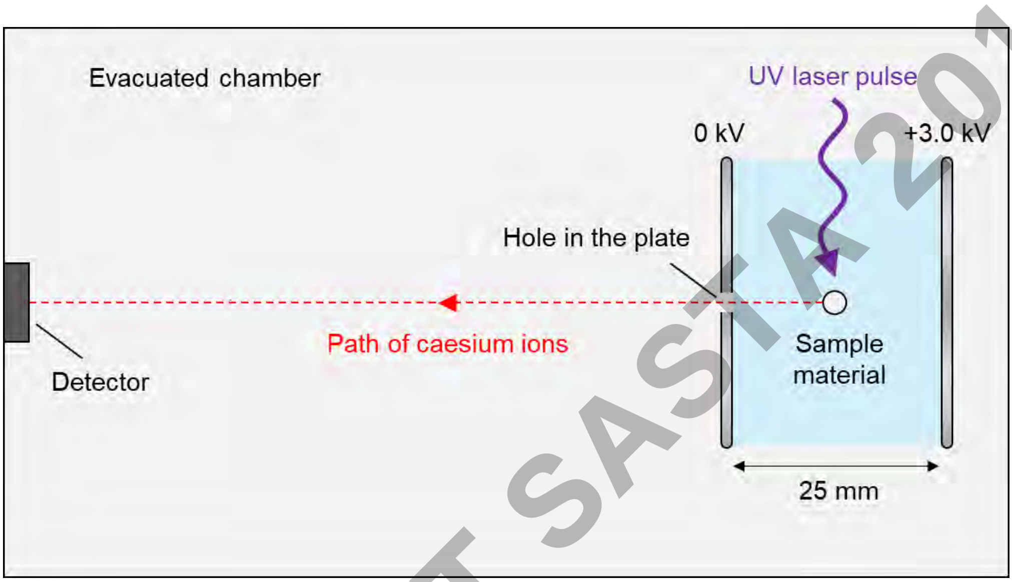
(1 mark) **IAE2**

Question 62

A laser ionisation mass spectrometer (LIMS) is a device that is used to measure the masses of atoms and molecules in a sample material.

A UV laser pulse is directed at the sample material which is positioned midway between two parallel plates that are 25 mm apart in a vacuum.

The laser pulse causes ionisation of the sample material and the positive ions are accelerated in the direction of a detector as shown below.



The potential difference between the two plates is 3.0 kV.

(a) Calculate the electric field strength between the two plates.

(2 marks) **KA4**

(b) The sample contains two isotopes of caesium.

Caesium-135 has a mass of 2.24×10^{-25} and caesium-137 has a mass of 2.27×10^{-25} kg.

The laser pulse produces caesium ions that have a charge of $+1.6 \times 10^{-19}$ C.

(1) Show that the acceleration of caesium-137 ions in the field between the plates is approximately 8.46×10^{10} m.s^{-2}.

(2 marks) **KA4**

(2) Calculate the magnitude and state the direction of the electrostatic force on caesium-137 in the field between the plates.

(3 marks) **KA4**

(3) Show that the time taken for caesium-137 ions to reach to the 0 kV plate after the sample is ionised by the laser pulse is 5.44×10^{-7} s.

(3 marks) **KA4**

(4) Calculate the velocity of caesium-137 ions upon reaching the 0 kV plate.

(2 marks) **KA4**

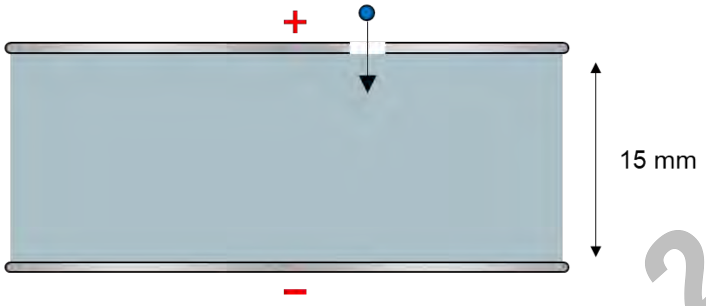
(5) State and explain which caesium ion reaches the detector first.

(3 marks) **KA2**

Question 63

An electron enters a uniform electric field through a gap between two parallel plates that are 15.0 mm apart in a vacuum.

The electron is initially moving with a velocity of 3.00×10^{6} m.s^{-1} when it enters the field through a gap in the positive plate as shown in the diagram below.



The vertical acceleration of the electron in the field is 5.85×10^{14} m.s^{-2}.

(a) Calculate the potential difference between the plates.

(4 marks) **KA4**

(b) Calculate the time it takes the electron to return to the positive plate.

(3 marks) **KA4**

(c) Calculate the distance the electron moves in the direction of the negative plate before returning to the positive plate.

(2 marks) **KA4**

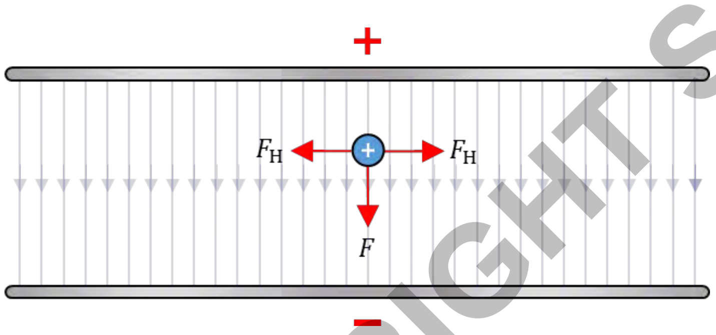
When a charged particle moves at an angle to the uniform electric field the component of the velocity perpendicular to the field remains constant.

- Compare the motion of a projectile in the absence of air resistance with the motion of a charged particle in a uniform electric field.
- Solve problems for the motion of charged particles that enter a uniform electric field perpendicular to the field.
- Solve problems for the motion of charged particles that enter a uniform electric field at an angle to the field where the displacement of the charged particle parallel to the field is zero.

A charged particle (q) experiences an electrostatic force (F) when it enters a uniform electric field (E).

$$F = Eq$$

The electrostatic force on the charge acts in the vertical direction in the electric field between two parallel plates. There is no horizontal component of the electric force (F_H) acting on a charge as it moves between the plates as shown below.



$$F_H = F\cos\theta$$
$$F_H = F\cos 90$$
$$F_H = 0$$

The motion of the charge in a uniform electric field is analogous to the motion of a projectile in a uniform gravitational field in the absence of air resistance as both are affected by vertical forces only (Figure 2.24).

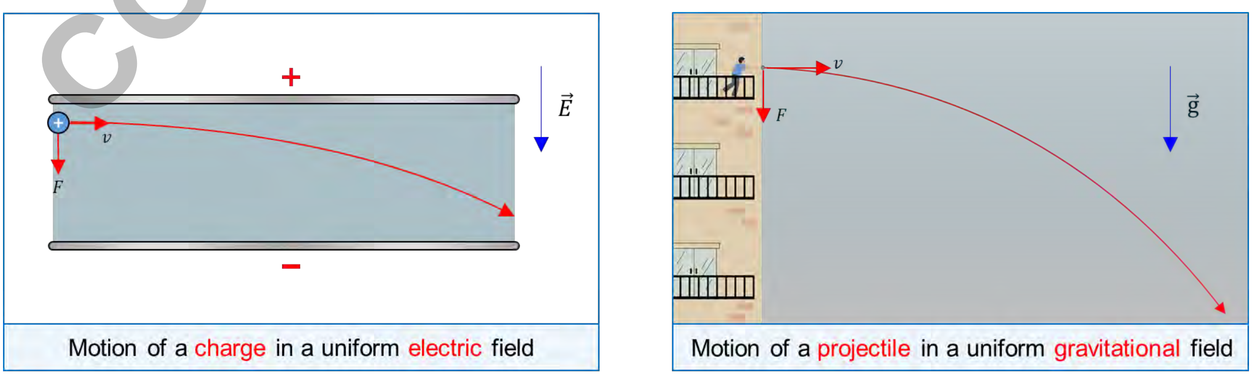


Figure 2.24: Parabolic motion of a charge (left) and projectile (right) in uniform fields.

$$E = \frac{\Delta V}{d}$$

$$E = \frac{1.0 \text{ x } 10^3}{0.050}$$

$$E = 2.0 \text{ x } 10^4 \ V.\text{m}^{-1}$$

$$a = \frac{Eq}{m}$$

$$a = \frac{2.0 \text{ x } 10^4 \text{ x } 1.6 \text{ x } 10^{-19}}{9.11 \text{ x } 10^{-31}}$$

$$a = 3.5 \text{ x } 10^{15} \text{ m.s}^{-2}$$

$$s_{\text{v}} = v_{\text{v}}t + \frac{1}{2}a_{\text{v}}t^2$$

$$s_{\text{v}} = (0 \text{ x } t) + \frac{1}{2}a_{\text{v}}t^2$$

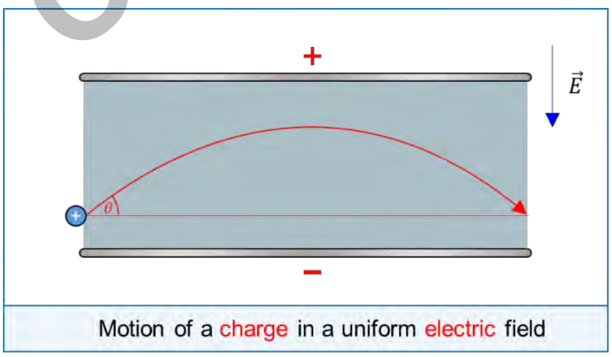
$$s_{\text{v}} = \frac{1}{2}a_{\text{v}}t^2$$

$$s_{\text{v}} = \frac{1}{2}(3.5 \text{ x } 10^{15})(2.4 \text{ x } 10^{-9})^2$$

$$s_{\text{v}} = 0.010 \text{ m} \quad (1.0 \text{ cm})$$

Motion at an angle to a uniform electric field

The motion of a charged particle entering at an angle to the field is also described using the equations of motion under constant acceleration. Calculations of the motion of charges entering at an angle to a uniform electric field in this course are limited to situations where the displacement parallel to the field is zero. This is analogous to uni-level projection in projectile motion (Figure 2.26).



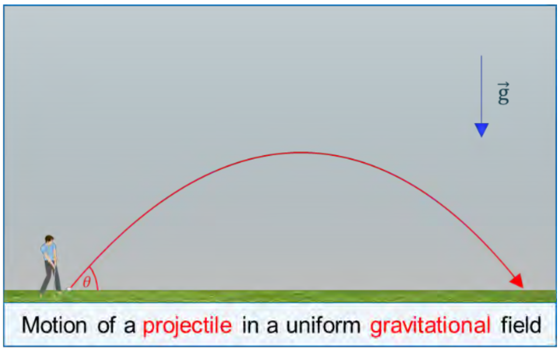


Figure 2.26: Parabolic motion of a charge (left) and a projectile (right) in uniform fields.

2.3: Magnetic Fields

Magnetic fields are associated with moving charges, such as charges in an electric current.

Current-carrying conductors produce magnetic fields; these fields are utilised in solenoids.

Magnetic field lines can be used to represent the magnetic field. The direction of the magnetic field depends on the direction of the moving charge that is producing the magnetic field.

The magnitude of magnetic field strength, B, at any point is represented by the number of lines crossing a unit area perpendicular to the field in the vicinity of the point.

- Sketch and/or interpret the magnetic field lines produced by an electric current flowing in a straight conductor, a loop, and a solenoid.

An electric field is a region of space surrounding point charges and charged objects. All electric charges and charged objects have an associated electric field regardless of whether they are stationary or in motion. However, a moving electric charge produces a second type of field called a **magnetic field**. A magnetic field is a region of space surrounding a charged particle or object that is in motion. Charged particles in motion experience a force when they enter the magnetic field of another moving charge.

Current-carrying conductors

Materials that contain free moving charged particles are called **electrical conductors**. A length of copper wire an example of an electrical conductor. The wire is made of copper atoms that contain free electrons (Figure 2.27).

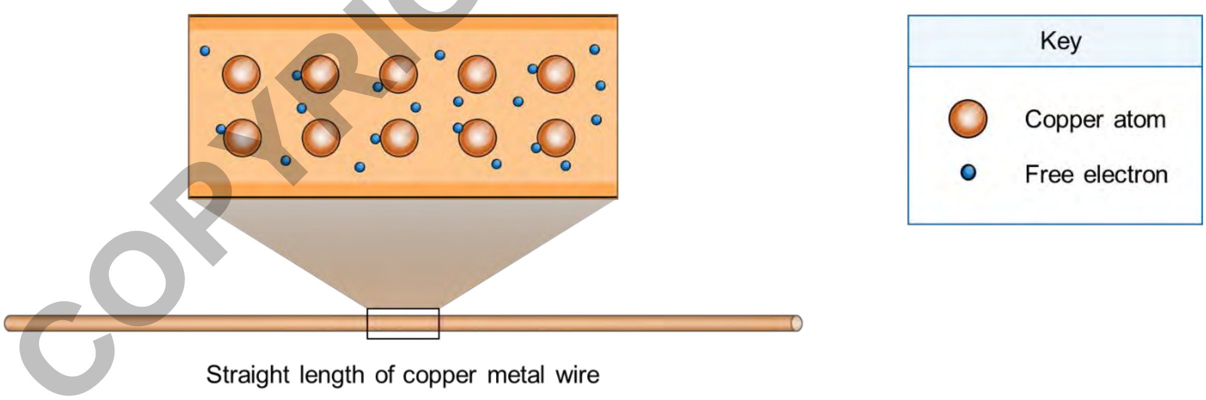


Figure 2.27: Straight length of copper metal wire (electrical conductor).

Free electrons in a metal wire flow as an electric current when the wire is connected to a source of electric potential difference such as a battery. The magnitude of electric current (I) flowing through a material is equal to the rate of flow of electric charge (q) and is calculated using the formula below.

$$I = \frac{q}{\Delta t}$$

The term electric current refers to the conventional current which is a flow of positive charge. The conventional current in a conductor is in the opposite direction to the electron current (flow of electrons).

The straight length of copper metal wire contains charges in motion. Charges in motion produce magnetic fields in the region around the wire. The magnetic field around a straight length of copper metal wire is depicted in Figure 2.28.

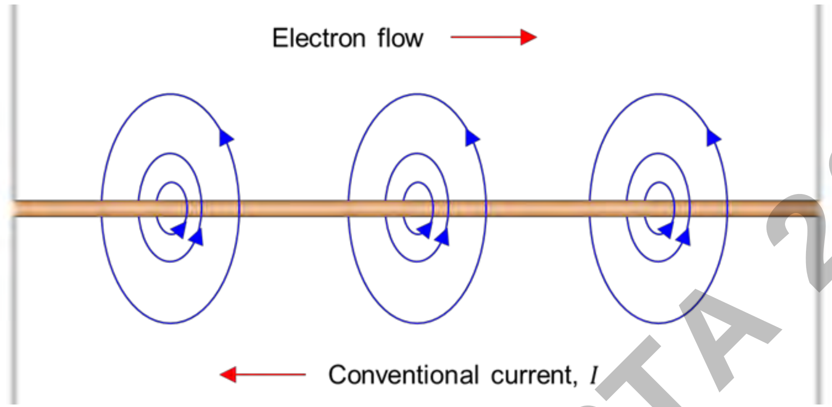


Figure 2.28: magnetic field around a straight length of copper metal wire.

The following properties of the magnetic field around a straight length of metal wire are identified:

- The magnetic field is circular in nature and is in the plane at right angles to the straight length of wire.
- The magnetic field is represented by concentric circles that become more widely spaced as the distance from the wire increases.
- The direction of the magnetic field lines is the direction of the force on a **compass needle** placed at that point (Figure 2.29).

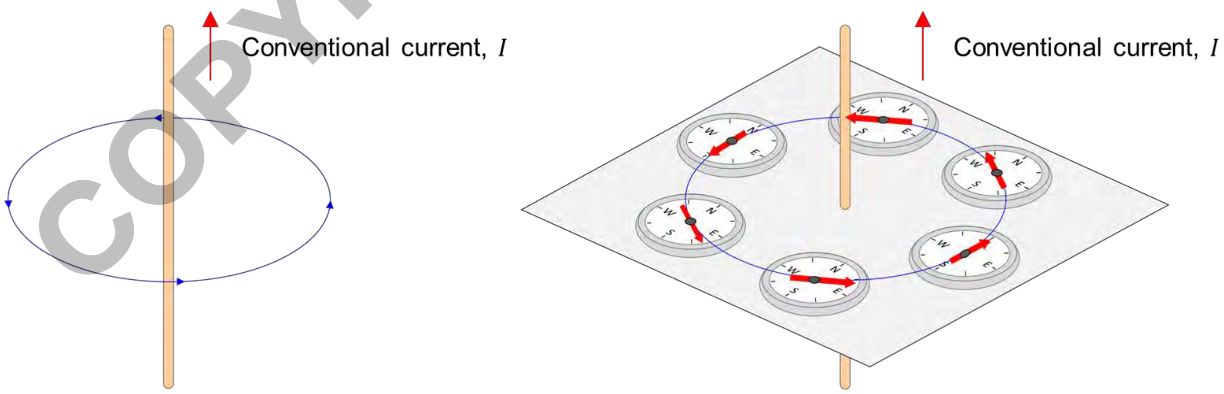


Figure 2.29: Direction of magnetic field around a straight length of current-carrying conductor.

Figure 2.29 shows that the compass needles are **tangent** to the circle around the wire indicating that the direction of the magnetic field is tangent to the circular field lines. It is important to note that magnetic field lines form closed loops that do not begin or end, and that field lines never cross.

The magnetic field around a straight length of current-carrying conductor such as a metal wire are also represented from the point-of-view of a person looking along the plane of the conductor. The diagrams in Figure 2.30 depict the view of the magnetic field to an observer that has an "end on" view looking along the length of the straight conductor.

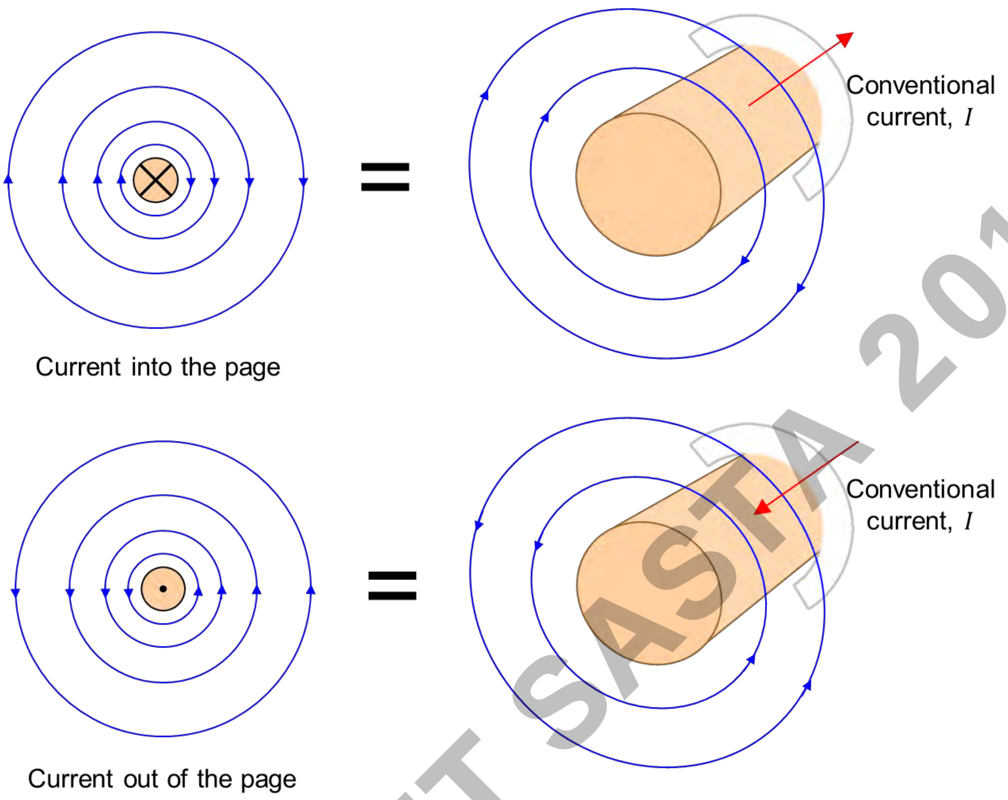


Figure 2.30: Magnetic field around a straight length of current-carrying conductor.

The direction of the current flow in an "end on" diagram is represented using a dot or a cross.

- A **dot** represents a current that is moving **out of the page**. The dot represents the pointed=tip of a vector arrow as if it were moving directly towards your eyes.
- A **cross** represents a current that is moving **into the page**. The cross represents the tail of a vector arrow as if it were moving away from your eyes.

Right-hand curl rule

The physical law stating that an electric current creates a magnetic field was discovered in 1820 by the Danish physicist Hans Christian Oersted (Figure 2.31). Oersted investigated the relationship between electric current and the resulting magnetic field and concluded that:

- magnetic field lines encircle a current-carrying conductor;
- magnetic field lines lie in a plane perpendicular to a current-carrying conductor;
- the strength of the magnetic field is directly proportional to the magnitude of the current and is inversely proportional to the distance from the conductor.

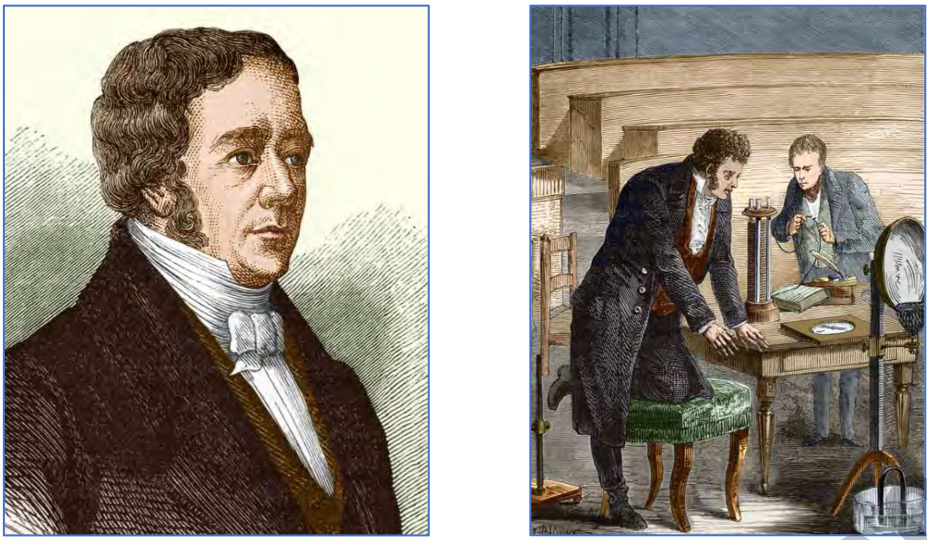


Figure 2.31: Hans Christian Oersted (1820).

Oersted discovered that the direction of the magnetic field at a point is the direction that a compass needle points, and can be found using the **right hand curl rule**. The right hand curl rule is a simple technique for determining the direction of the magnetic field around an electrical conductor.

If the right hand is wrapped around the conductor so the thumb points in the direction of the conventional current, the fingers will curl around the conductor in the direction of the magnetic field as shown in Figure 2.32.

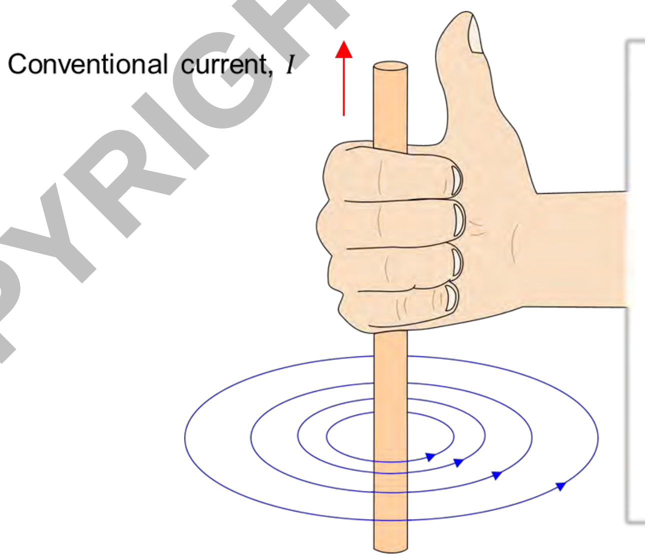


Figure 2.32: The right hand curl rule for a straight conductor.

It is important to keep the thumb in the plane of the conductor and to curl the fingers and not the wrist when determining the direction of the magnetic field. It is also worth noting that in practice, you would never place your hand on a conductor that is carrying a current as depicted in Figure 2.32 as this is action is dangerous.

Magnetic Field of a Current Loop

An electrical conductor may also be shaped into a loop that bends round and crosses itself. Loops may be circular, square, rectangular or any other closed two-dimensional shape. An electric current flowing through a loop produces a magnetic field in the region surrounding the conductor.
Figure 2.33 shows the magnetic field around a circular loop conductor.

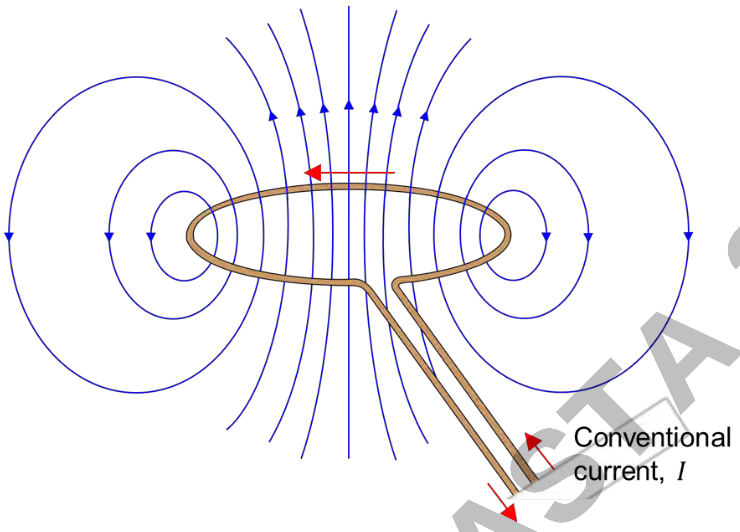


Figure 2.33: Magnetic field around a circular loop conductor.

The right hand curl rule is used to determine the direction of the magnetic field. The loop is grasped with the right hand and the thumb is extended in the direction of the conventional current flow through the conductor. The fingers then curl around the conductor giving the direction of the magnetic field around the conductor as depicted in Figure 2.34.

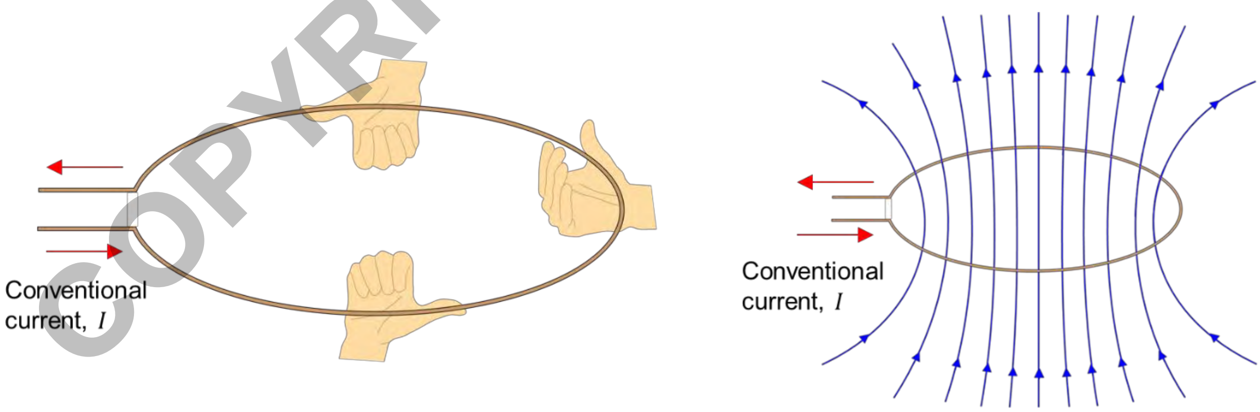


Figure 2.34: Magnetic field around a circular loop conductor.

Note that the magnetic field lines go straight through the circular loop conductor and join up to form closed loops as depicted in Figure 2.33. The field through the loop is approximately uniform, as shown by the uniform spacing and symmetrical distribution of field lines through the loop.

Magnetic Field of a Solenoid

A solenoid is a long cylindrical coil of conducting wire that is wound into a tightly packed helix (Figure 2.35).

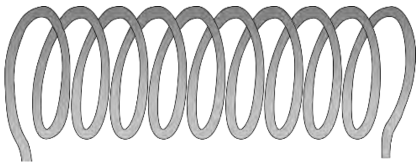


Figure 2.35: Example of a solenoid

An electric current flowing through the coils produces a magnetic field through, and in the region surrounding the solenoid. The magnetic field inside the coil is approximately uniform as depicted by the even distribution of magnetic field lines within the coil (Figure 2.36).

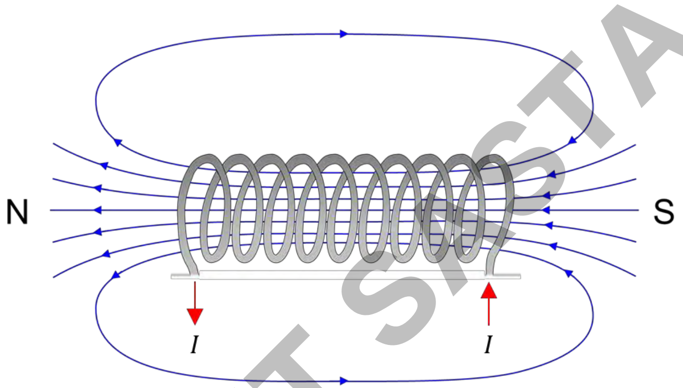


Figure 2.36: Magnetic field around a solenoid.

One end of a solenoid is a **magnetic north pole (N)** and the other is a **magnetic south pole (S)**. Note that magnetic field lines are directed towards the magnetic north pole and away from the magnetic south pole. The **right hand solenoid rule** is used to determine the direction of the magnetic north pole of a solenoid. The solenoid is grasped with the right hand with the fingers wrapped around the coil in the direction of the current. The outstretched thumb gives the direction of the magnetic north pole of the solenoid (Figure 2.37).

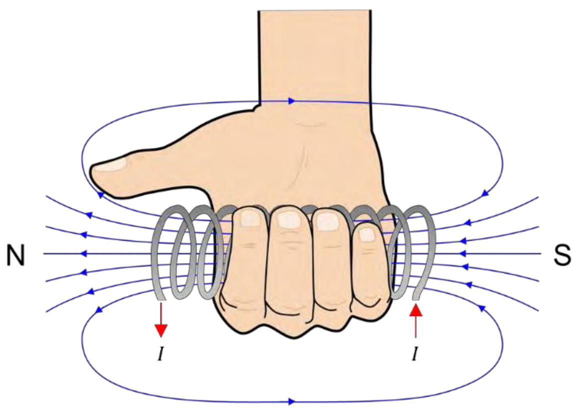


Figure 2.37: Right hand solenoid rule

A solenoid may also be represented by an "end on" diagram showing the direction of current flow in the coils of a solenoid (Figure 2.38).

Figure 2.38: "end on" diagram showing the direction of current flow in the coils of a solenoid

The direction of the magnetic field can be determined using the right hand rule or the right hand solenoid rule.

- If the right hand rule is used, the thumb points in the direction of the conventional current flow and the curl of the fingers gives the direction of the magnetic field.
- If the right hand solenoid rule is used, the fingers point in the direction of the conventional current flow and the extended thumb points in the direction of the magnetic north pole of the solenoid.

The magnetic field around the solenoid in an "end on" diagram is shown in Figure 2.39.

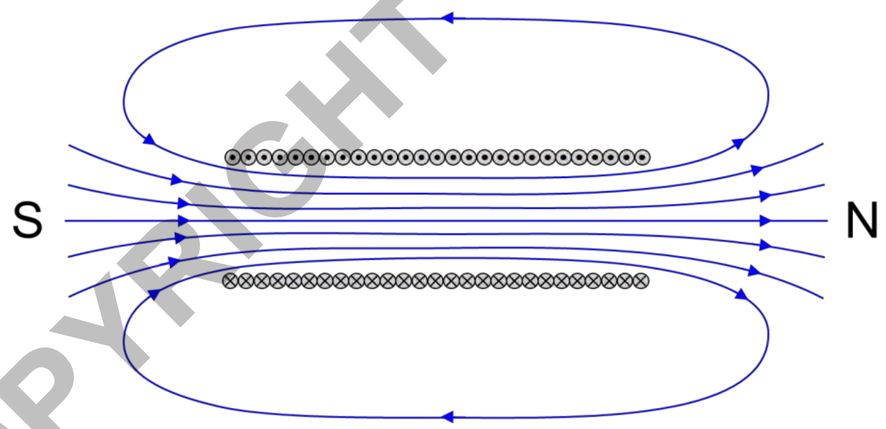


Figure 2.38: "end on" diagram showing the direction of current flow in the coils of a solenoid

Magnetic Field Strength

Magnetic field strength, $\vec{B}$, is a measure of the force acting on a moving charge. For a current-carrying conductor, magnetic field strength is a measure of the force acting on the current flowing along the length of the conductor. The magnetic field strength at any point around a current-carrying conductor is represented by the number of lines crossing a unit area. The magnetic field strength is greater in regions where there are a greater number of field lines per unit area.

The magnitude of the magnetic field strength in the vicinity of a current carrying conductor is given by $B = \frac{\mu_0 I}{2\pi r}$ where r is the radial distance to the conductor.

- Solve problems involving the use of $B = \frac{\mu_0 I}{2\pi r}$

The strength of the magnetic field (B) surrounding a current carrying conductor is calculated using the formula below. The SI unit of magnetic field strength is the tesla (T) and is named for the Serbian-American physicist and inventor, Nikola Tesla.

Formula	$B = \frac{\mu_0 I}{2\pi r}$

Symbol	Variable	SI unit
B	Magnetic field strength	(T)
I	Magnitude of current flowing in the conductor	(A)
r	Radial distance from the conductor	(m)
μ_0	Permeability of free space	($T.m.A^{-1}$)

The permeability of free space (μ_0) is a constant of proportionality in the formula used to calculate the magnetic field strength of a current-carrying conductor. The permeability of free space is approximately $4\pi \times 10^{-7}$ $T.m.A^{-1}$.

Example 1:

A straight length of copper wire carries a current of 1.5A.

Calculate the magnetic field strength at a radial distance of 50 cm away from and in the plane at right angles to the wire.

B	=	$\frac{\mu_0 I}{2\pi r}$
B	=	$\frac{4\pi \times 10^{-7} \times 1.5}{2\pi(0.50)}$
B	=	6.0×10^{-7} T

Example 2:

A transmission power line carries a current of 500 A.

How far from the power line is the magnetic field strength 2.2×10^{-5} T?

r	=	$\frac{\mu_0 I}{2\pi B}$
r	=	$\frac{4\pi \times 10^{-7} \times 500}{2\pi(2.2 \times 10^{-5})}$
r	=	4.5 m

Question 67

Current flows in a length of copper wire that is connected to a power supply.

A diagram of the length of wire is given below.

(a) Sketch the magnetic field around the wire. (3 marks) **KA1**

(b) An "end on" diagram of the copper wire is given below.

Sketch the magnetic field around the wire. (3 marks) **KA1**

(c) A current of 0.50 A flows in the wire.

Calculate the magnetic field strength at a radial distance of 15 cm from the wire.

(2 marks) **KA4**

(d) A Hall probe is a device used to measure the magnetic field strength around a conductor.

The probe measures a magnetic field strength of 0.25 μT at a radial distance r from the copper wire.

Calculate the value of r.

(3 marks) **KA4**

Direction of the magnetic force on a current-carrying conductor

The direction of the magnetic force on a current-carrying conductor is found using the **right hand palm rule**. The right hand palm rule for the magnetic force on a current-carrying conductor is summarised below:

- The thumb points in the direction of the conventional current flow;
- The fingers point in the direction of the magnetic field;
- The open palm pushes in the direction of the magnetic force on the conductor (Figure 2.42).

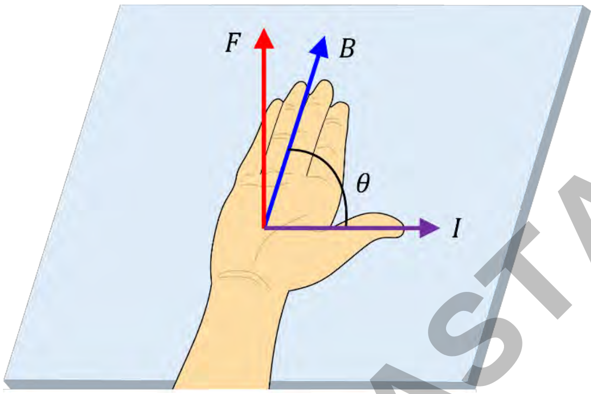


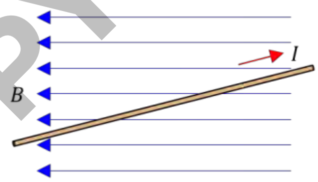
Figure 2.42: Right hand palm rule for the direction of the magnetic force.

Question 69

The right hand palm rule is used to determine the direction of the magnetic force on a current-carrying conductor that is inside a uniform magnetic field of strength B.

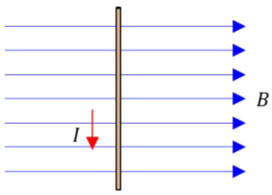
Use the right hand palm rule to determine the direction of the magnetic force in the examples below.

(a)



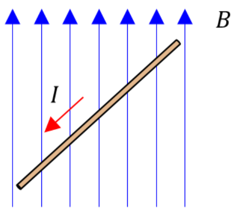
Direction: ________________

(b)



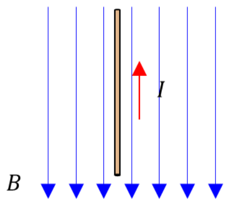
Direction: ________________

(c)



Direction: ________________

(d)



Direction: ________________

(4 marks) **KA1**

A charged particle moving at right angles to a uniform magnetic field experiences a force of constant magnitude at right angles to the velocity. The force changes the direction but not the speed of the charged particle and hence it moves with uniform circular motion.

- Explain how the velocity dependence of the magnetic force on a charged particle causes the particle to move with uniform circular motion when it enters a uniform magnetic field at right angles.
- Derive $r = \frac{mv}{Bq}$ for the radius r of the circular path of an ion of charge q and mass m that is moving with speed v at right angles to a uniform magnetic field of magnitude B.
- Solve problems involving the use of $r = \frac{mv}{Bq}$

A charged particle moving perpendicular to a uniform magnetic field experiences a magnetic force (F) perpendicular to its direction of motion (v) as shown in Figure 2.43 below.

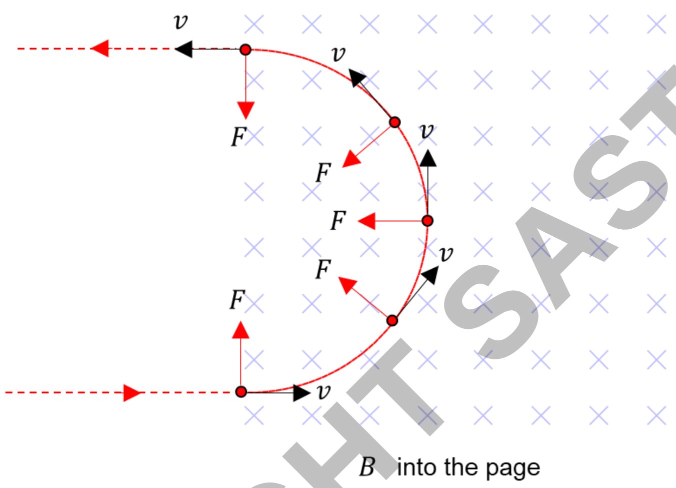


Figure 2.43: Motion of a positive charge inside a uniform magnetic field.

The magnetic force causes the charge to move with uniform circular motion inside the magnetic field. The magnetic force ($F = qvB$) causing the centripetal acceleration $\left(\frac{mv^2}{r}\right)$ of the charge.

$$\begin{aligned} F &= \frac{mv^2}{r} \\ qvB &= \frac{mv^2}{r} \end{aligned}$$

A formula used to calculate the radius of the circular path of the charged particle is derived below.

$$\begin{aligned} qvB &= \frac{mv^2}{r} \\ q\cancel{v}B &= \frac{mv^{\cancel{2}}}{r} \\ qB &= \frac{mv}{r} \\ r &= \frac{mv}{qB} \end{aligned}$$

Cyclotrons are used to accelerate ions to high speed. The high-speed ions are collided with other nuclei to produce radioisotopes that can be used in medicine and industry.

In a cyclotron, the electric field in the gap between the dees increases the speed of the charged particles.

- Describe how an electric field between the dees can transfer energy to an ion passing between them.
- Describe how ions could be accelerated to high energies if they could be made to repeatedly move across an electric field.
- Calculate the energy transferred to an ion each time it passes between the dees.
- Explain why the ions do not gain kinetic energy when inside the dees.

A **cyclotron** is a type of particle accelerator that is used in the production of high energy charged particles. The high energy particles produced in a cyclotron are used in the production of radioactive isotopes that are used in nuclear medicine and industry. The structural features of a cyclotron are identified in Figure 2.44.

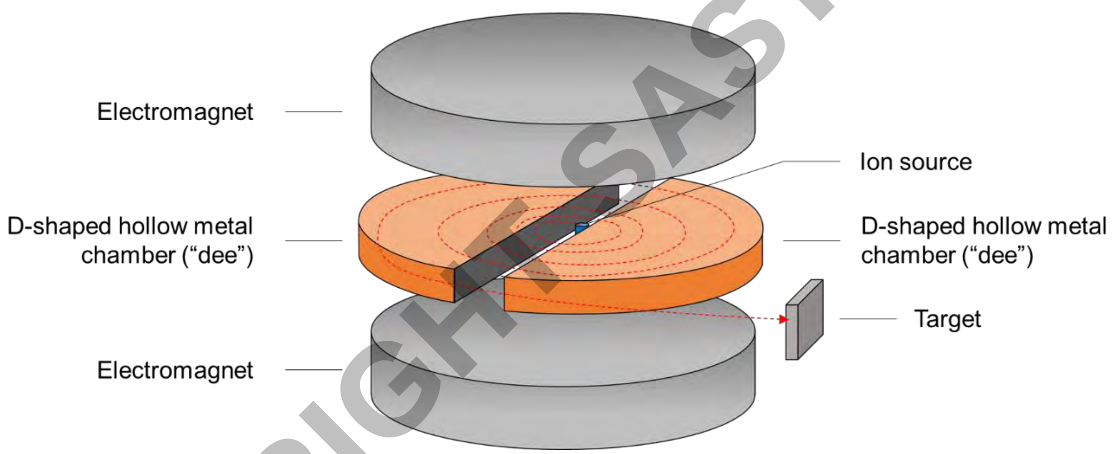


Figure 2.44: Some structural features of a cyclotron.

Feature	Description
Ion source	The source of electrically charged particles (ions) that are accelerated in the cyclotron. Typically, hydrogen ions or deuterons (deuterium ions) are produced in the ion source.
Dees	Two hollow semicircular (D-shaped) chambers composed of a non-magnetic metal. An alternating potential difference is applied between the dees which produces an electric field. The field transfers energy to the ions as they cross the gap between the dees.
Electromagnet	Two large electromagnets create a permanent magnetic field that passes straight through the dees.
Target	The accelerated ions collide with the target producing radioactive isotopes that are used in nuclear medicine, industry or research.

Electric field in a cyclotron

An alternating potential difference (ΔV) is applied between the dees of a cyclotron. The alternating potential difference produces an electric field between, but not inside the dees (Figure 2.45).

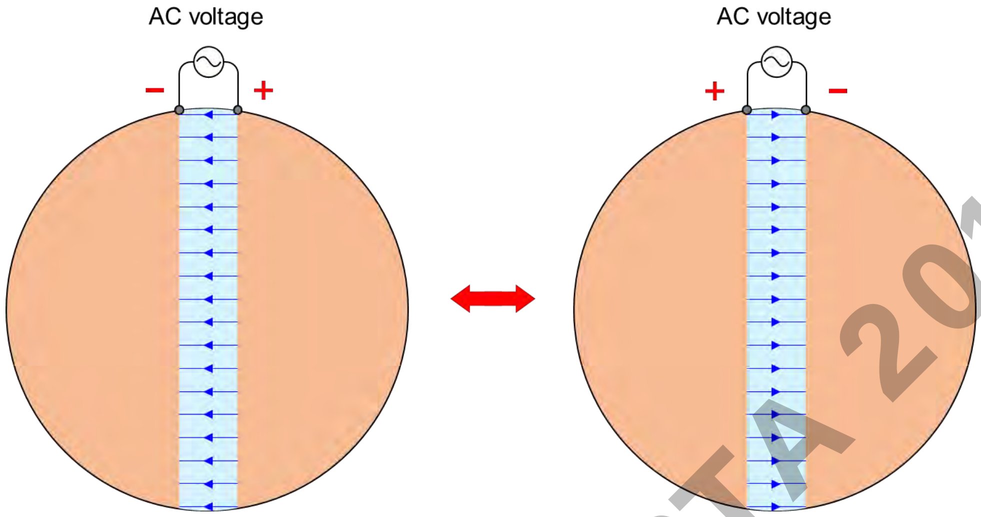


Figure 2.45: Alternating potential difference across the gap between the dees.

The ions enter the cyclotron from the ion source and are immediately accelerated by the electric field between the dees. The electric field transfers energy which increases the speed and kinetic energy of the ions. The magnitude of energy gained by the ions is equal to the amount of work (W) done by the field and is calculated using the formula below.

$$W = q\Delta V$$

The ions gain energy each time they cross the gap between the dees. The dees are hollow conductors with no internal electric field. For this reason, the ions gain no energy inside the dees.

Example:

A deuteron ($q = +1.6 \times 10^{-19}$ C) enters a cyclotron in the gap between the dees.

The potential difference between the dees in 50 kV.

Calculate the energy transferred to the deuteron each time it crosses the gap between the dees.

$$W = q\Delta V$$
$$W = 1.6 \times 10^{-19} \times 50 \times 10^{3}$$
$$W = 8.0 \times 10^{-15}\ \text{J}$$

The magnetic field within the dees of a cyclotron causes the charged particles to travel in a circular path, so that they repeatedly pass through the electric field.

- Describe the nature and direction of the magnetic field needed to deflect ions into a circular path in the dees of a cyclotron.
- Derive the formula $T = \frac{2\pi m}{Bq}$ for the period T of the circular motion of an ion, and hence show that the period is independent of the speed of the ion.
- Derive the formula $E_k = \frac{q^2B^2r^2}{2m}$ for the kinetic energy E_k of the ions emerging at radius r from a cyclotron.
- Use the formula $E_k = \frac{q^2B^2r^2}{2m}$ to show that E_k is independent of the potential difference across the dees and, for given ions, depends only on the magnetic field and the radius of the cyclotron.
- Solve problems involving the use of $T = \frac{2\pi m}{Bq}$ and $E_k = \frac{q^2B^2r^2}{2m}$
- Discuss the importance of being able to generate radioisotopes in a timely manner near the location they are required.

The electromagnets of a cyclotron create a magnetic field between the dees (Figure 2.46).

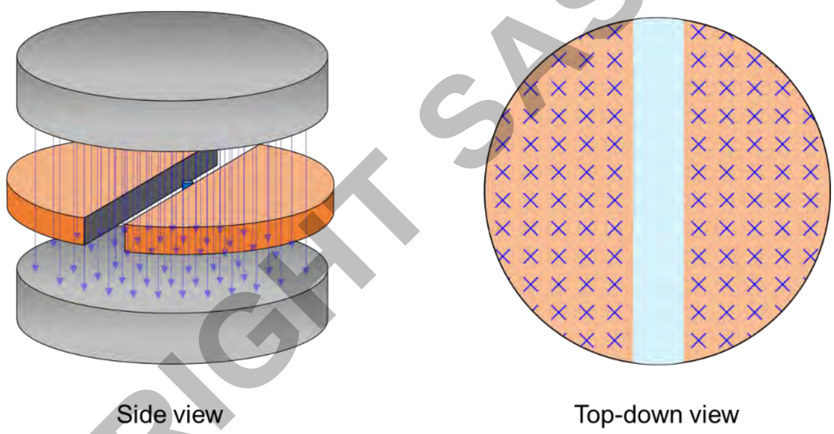


Figure 2.46: Magnetic field of the cyclotron.

The magnetic field is at right angles to the direction of motion of the ions inside the dees. The ions experience a magnetic force at right angles to their direction of motion upon entering the magnetic field inside the dees. The magnetic force causes the ions to move in a semicircular path (Figure 2.47).

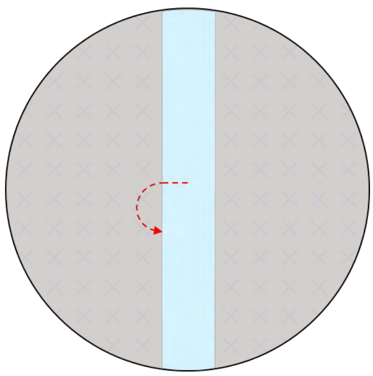


Figure 2.47: Semicircular path of a positively charged ion inside the dees.

The magnetic force causes the ions to move in a semicircular path and return to the gap between the dees. The potential of the two terminals is alternated when the ions return to the electric field between the dees. The potential must alternate for the ions to move in the direction of the electrostatic force in the electric field between the dees (Figure 2.48).

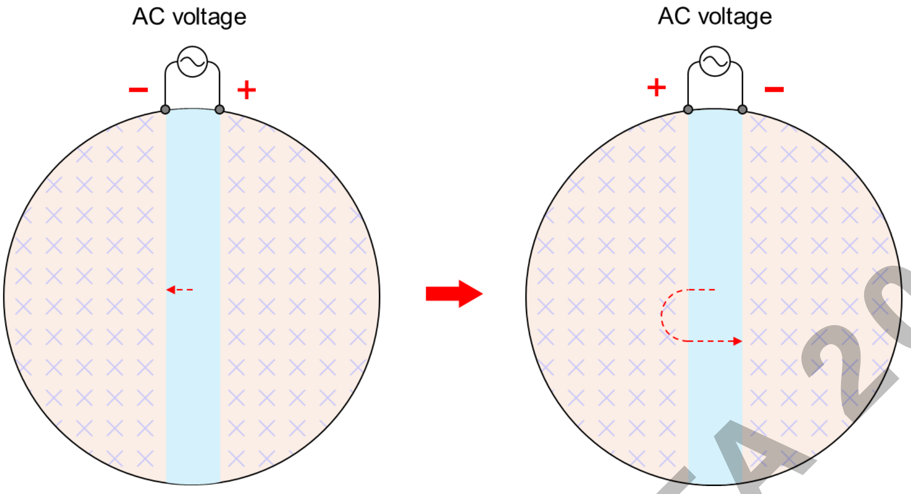


Figure 2.48: Positively charged ion cross the gap between the dees for a second time.

The speed (v) of the ions increases as the ions cross the gap between the dees. The increase in speed results in a proportional increase in the radius (r) of the semicircular path of the ions as they enter the uniform magnetic field inside the dees.

$$r = \frac{mv}{qB}$$

$$r \propto v$$

Figure 2.49 shows the increase in radius of the semicircular path of ions entering the dee after crossing the electric field for the second time.

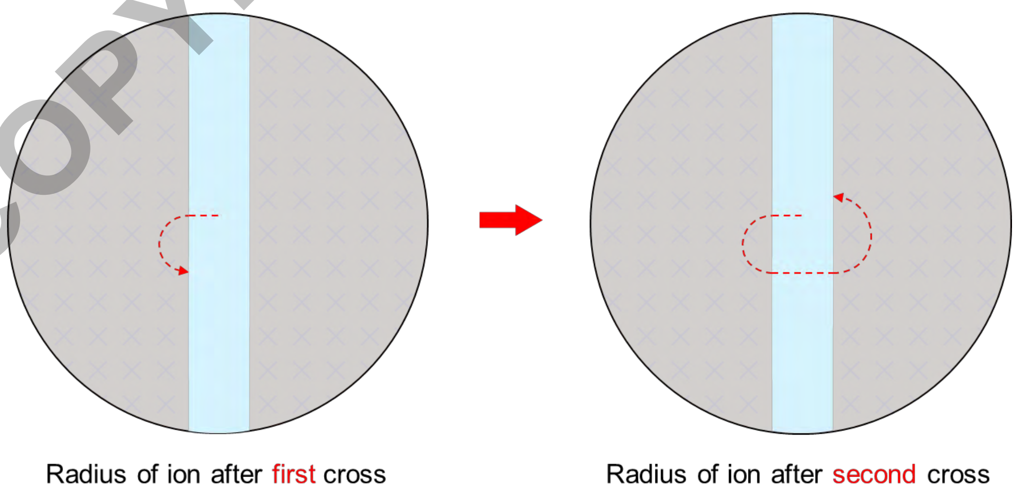


Figure 2.49: Increase in radius of semicircular path of an ion inside a cyclotron.

Period of circular motion

The period of circular motion of the ions inside the cyclotron is constant. A formula for calculating the period of one full revolution of an ion inside the cyclotron is derived below.

$$T = \frac{2\pi r}{v} \qquad v = \frac{rqB}{m}$$

$$T = \frac{2\pi r}{rqB/m}$$

$$T = \frac{2\pi rm}{rqB}$$

$$T = \frac{2\pi \cancel{r}m}{\cancel{r}qB}$$

$$T = \frac{2\pi m}{qB}$$

The formula shows that the period of circular motion of the ions is independent of their speed (v) meaning that the time taken for the ions to complete a half or full revolution of the cyclotron is constant. This property of cyclotron motion is important as it allows an operator to set the frequency (f) of the alternating potential ahead of time.

An operator sets the frequency at which the potential alternates between positive and negative to allow ions to accelerate across the electric field in the gap between the dees. The frequency of the alternating potential is calculated using the formula below.

$$f = \frac{1}{T} \qquad T = \frac{2\pi m}{qB}$$

$$f = \frac{1}{2\pi m/qB}$$

$$f = \frac{1qB}{2\pi m}$$

$$f = \frac{qB}{2\pi m}$$

The formula reveals that the frequency of the alternating potential is constant as the charge (q), mass (m) and magnetic field strength (B) are all constant.

Kinetic energy of the emerging ions

The speed and radius of the ions increases each time they cross the gap between the dees. The ions emerge from the cyclotron when the radius of the circular path is approximately equal to the internal radius of the cyclotron (Figure 2.50).

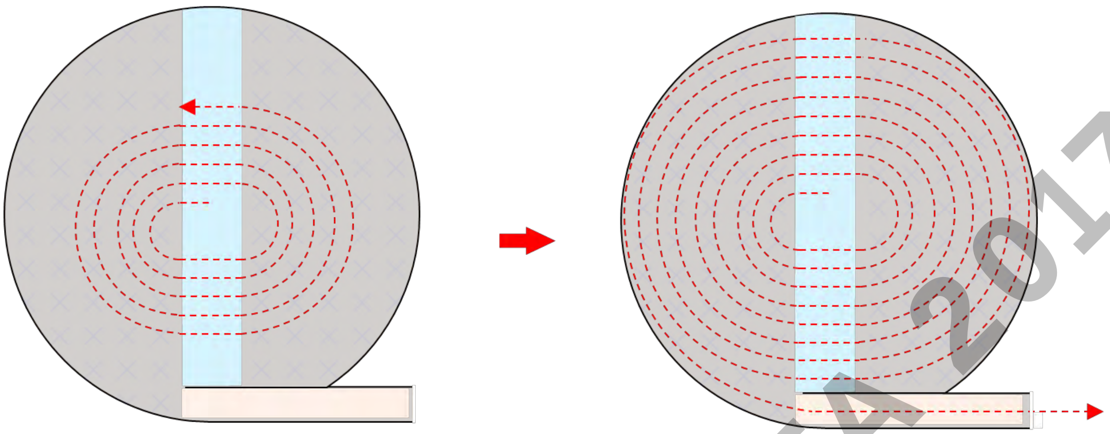


Figure 2.50: Ions emerging from the cyclotron.

The kinetic energy (E_k) of the ions increases each time they cross the gap between the dees. The kinetic energy of the ions emerging from the cyclotron is calculated using the formula derived below.

$$E_k = \frac{1}{2}mv^2 \qquad v = \frac{rqB}{m}$$

$$E_k = \frac{1}{2}m\left(\frac{rqB}{m}\right)^2$$

$$E_k = \frac{1}{2}m\left(\frac{r^2q^2B^2}{m^2}\right)$$

$$E_k = \frac{mr^2q^2B^2}{2m^2}$$

$$E_k = \frac{\cancel{m}r^2q^2B^2}{2m^{\cancel{2}}}$$

$$E_k = \frac{r^2q^2B^2}{2m}$$

The formula reveals that the kinetic energy of the emerging ions is only dependent on the radius of the circular path (r) of the emerging ions which is approximately equal to the internal radius of the cyclotron. The kinetic energy of the emerging ions is independent of the potential difference between the dees.

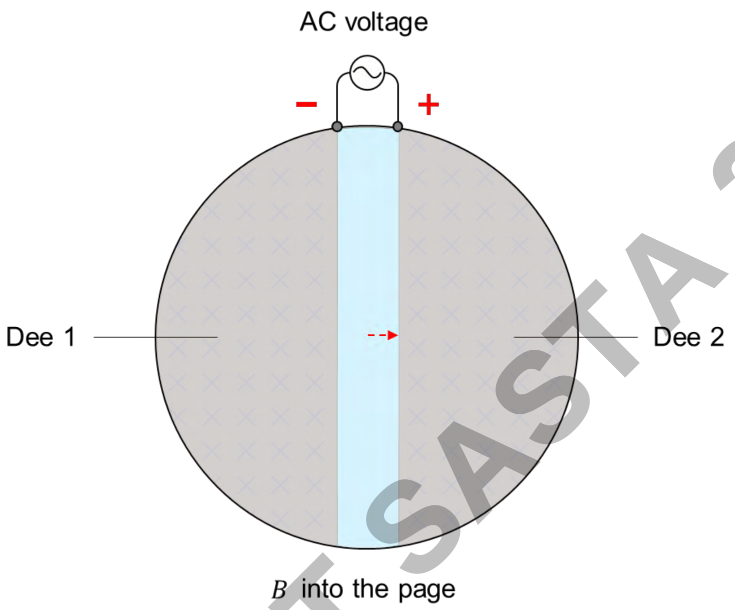
Question 76

A cyclotron is a device used to accelerate charged particles to high speeds.

One type of cyclotron used accelerates negatively charged hydride ions (H^-).

Hydride ions have a mass of 1.675×10^{-27} kg and a charge of -1.6×10^{-19} C.

Hydride ions are introduced into the cyclotron from the ion source located between the dees as shown in the diagram below.



(a) State the direction of the magnetic force on the hydride ions entering Dee 2.

____ (1 mark) **KA1**

(b) Describe and explain the motion of a hydride ion inside the dees.

____

____

____

____

____

____ (3 marks) **KA1**

(c) State the function of the electric field between the dees.

____

____ (1 mark) **KA1**

(d) The hydride ions are accelerated in a cyclotron that has the following properties.

Property	Magnitude
Alternating potential	33 kV
Magnetic field strength	2.35 T
Kinetic energy of emerging hydride ions	12×10^6 eV

(1) Calculate the energy transferred to a hydride ion each time it crosses the gap between the dees.

(2 marks) **KA4**

(2) Calculate the time a hydride ion spends inside Dee 2.

(3 marks) **KA4**

(3) Calculate the diameter of the cyclotron.

(5 marks) **KA4**

(e) The alternating potential between the dees is decreased to 25 kV.

State and explain the effect of this change on the kinetic energy of the emerging hydride ions.

____

____

____ (2 marks) **KA1**

(f) Electrons are removed from hydride ions before being focussed onto a target.

The removal of electrons from hydride ions produces protons.

Protons are collided with a target producing radioisotopes.

Fluorine-18 is a radioisotope produced by colliding protons with an oxygen-18 target.

State why it is necessary for protons to have high kinetic energies before being collided with oxygen-18.

(1 mark) KA2

(g) The South Australian Health and Medical Institute (SAHMRI) houses a cyclotron that is used in the production of medical radioisotopes including fluorine-18.

Fluorine-18 has a half-life of less than two hours.

(1) Explain the advantage of the SAHMRI being located next to the new Royal Adelaide Hospital (RAH).

(2 marks) KA3

(2) Before the SAHMRI was constructed, fluorine-18 and other radioisotopes had to be transported to Adelaide by aeroplane.

State two disadvantages to patients in Adelaide awaiting treatments requiring fluorine-18.

(2 marks) KA3

42	(a)	$t = \frac{1}{\sqrt{1-v^2/c^2}}t_0$ $t_0 = t \times \sqrt{1-v^2/c^2}$ $t_0 = 62.3 \times 10^{-9} \times \sqrt{1-\frac{(0.98 \times 3.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}$ $t_0 = 1.24 \times 10^{-8}$ s $t_0 = 12.4$ ns	 1 1 1 1
	(b)	A 50% reduction in the original number of kaons occurs in one half-life (62.3 ns in the reference frame of the detectors). The minimum distance between detectors must be greater than the distance travelled by 50% of the particles (s). $s = vt$ $s = (0.98 \times 3.00 \times 10^8)(62.3 \times 10^{-9})$ $s = 18.3$ m	 1 1
	(c)	The Special Theory of Relativity predicts that time is dilated in the reference frame of the detectors to ensure that the speed of light remains constant. The kaons have a longer half-life in the reference frame of the detectors compared the reference frame of the kaons due to time dilation.	1 1
43	(a)	0.42 m. This is the proper length (l_0) of the picture as the length is measured in the reference frame where the picture and observer (astronaut) are at rest with respect to each other.	1 1
	(b)	$l = \frac{l_0}{\gamma}$ $l = l_0 \times \sqrt{1-(v^2/c^2)}$ $l = 0.42 \times \sqrt{1-\frac{(0.85 \times 3.0 \times 10^8)^2}{(3.0 \times 10^8)^2}}$ $l = 0.22$ m	 1 1 1
	(c)	0.35 m The width of the picture is unchanged as the length of the picture is only contracted in the direction of motion of the spacecraft.	1 1
44	(a)	$t = \frac{l_0}{v}$ $t = \frac{3.20 \times 10^3}{(0.995 \times 3.00 \times 10^8)}$ $t = 1.07 \times 10^{-5}$ s	 1 1
	(b)	$l = \frac{l_0}{\gamma}$ $l = l_0 \times \sqrt{1-(v^2/c^2)}$ $l = 3.20 \times 10^3 \times \sqrt{1-\frac{(0.995 \times 3.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}$ $l = 320$ m (319.6)	 1 1 1
	(c)	$t_0 = \frac{l}{v}$ $t_0 = \frac{319.6}{(0.995 \times 3.00 \times 10^8)}$ $t_0 = 1.07 \times 10^{-6}$ s	 1 1
	(d)	The time interval (t) between events is greater/time progresses more slowly/time is dilated in the reference frame of the linac which is moving relative to the electrons.	1

			Alternatively: Time in the reference frame of the electrons (proper time, t_0) is shorter as the electrons are stationary relative to the linac.	
45	(a)		Earth leaves the spaceship and moves away from the spacecraft at 0.75c.	1
			Mars moves uniformly in the direction of the spacecraft at 0.75c until it arrives at the same position as the spacecraft.	1
	(b)	(1)	$t = \frac{l_0}{v}$	
			$t = \frac{54.6 \text{ x } 10^9}{(0.75 \text{ x } 3.00 \text{ x } 10^8)}$	1
			$t = 243 \text{ s}$	1
		(2)	$l = \frac{l_0}{\gamma}$	
			$l = l_0 \text{ x } \sqrt{1 - (v^2/c^2)}$	1
			$l = 54.6 \text{ x } 10^9 \text{ x } \sqrt{1 - \frac{(0.75 \text{ x } 3.00 \text{ x } 10^8)^2}{(3.00 \text{ x } 10^8)^2}}$	1
			$l = 35.7 \text{ x } 10^9 \text{ m}$	1
		(3)	$t_0 = \frac{l}{v}$	
			$t_0 = \frac{35.7 \text{ x } 10^9}{(0.75 \text{ x } 3.00 \text{ x } 10^8)}$	1
			$t_0 = 159 \text{ s}$	1
46	(a)		$m = m_0\gamma$	
			$m = m_0 \text{ x } \frac{1}{\sqrt{1 - (v^2/c^2)}}$	
			$m = 9.11 \text{ x } 10^{-31} \text{ x } \frac{1}{\sqrt{1 - \frac{(0.995 \text{ x } 3.00 \text{ x } 10^8)^2}{(3.00 \text{ x } 10^8)^2}}}$	1
			$m = 9.12 \text{ x } 10^{-30} \text{ kg}$	1
	(b)		$p = mv$	
			$p = (9.12 \text{ x } 10^{-30}) \text{ x } (0.995 \text{ x } 3.00 \text{ x } 10^8)$	1
			$p = 2.72 \text{ x } 10^{-21} \text{ kg. m. s}^{-1}$	1
	(c)		$\frac{v^2}{c^2} = 1 - \left(\frac{m_0}{m}\right)^2$	
			$\frac{v^2}{c^2} = 1 - \left(\frac{9.11 \text{ x } 10^{-31}}{9.12 \text{ x } 10^{-30}}\right)^2$	
			$\frac{v^2}{c^2} = 0.99$	1
			$v^2 = 0.99 \text{ x } c^2$	1
			$v = \sqrt{0.99 \text{ x } (3.00 \text{ x } 10^8)^2}$	
			$v = 2.99 \text{ x } 10^8 \text{ m. s}^{-1}$	1
			$v = \frac{2.99 \text{ x } 10^8}{c}$	
			$v = \frac{2.99 \text{ x } 10^8}{3.00 \text{ x } 10^8}$	
			$v = 0.995c$	1
	(d)		The mass of protons and electrons increases to near infinite as they approach the speed of light;	1
			A particle with a near infinite mass requires a near infinite force/amount of energy to accelerate it further towards the speed of light which is not possible.	1

Question	Part	Sub-part	Answer	Mark
47	(a)	(1)	$t = \frac{l_0}{v}$	
			$t = \frac{0.30}{(0.30 \times 3.00 \times 10^8)}$	1
			$t = 3.33 \times 10^{-9}$ s	1
		(2)	$t = \frac{1}{\sqrt{1 - v^2/c^2}} t_0$	
			$t_0 = t \times \sqrt{1 - v^2/c^2}$	1
			$t_0 = 3.33 \times 10^{-9} \times \sqrt{1 - \frac{(0.30 \times 3.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}$	1
			$t_0 = 3.18 \times 10^{-9}$ s	1
	(b)		$l = vt_0$	
			$l = (0.30 \times 3.00 \times 10^8) \times (3.18 \times 10^{-9})$	1
			$l = 0.29$ m (0.289)	1
	(c)	(1)	$m = m_0 \gamma$	
			$m = m_0 \times \frac{1}{\sqrt{1 - (v^2/c^2)}}$	
			$m = 9.11 \times 10^{-31} \times \frac{1}{\sqrt{1 - \frac{(0.30 \times 3.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}}$	1
			$m = 9.55 \times 10^{-31}$ kg	1
		(2)	$p = mv$	
			$p = (9.55 \times 10^{-31}) \times (0.30 \times 3.00 \times 10^8)$	1
			$p = 8.60 \times 10^{-23}\ \text{kg.m.s}^{-1}$	1
	(d)		The angle of deflection (θ) necessary to produce the image correctly is dependent on the speed (v_1) of the electrons as shown in the formula $\theta = \arctan \frac{v_1}{v_2}$	1
			The speed (v) of electrons in the CRT is dependent on the mass of the electron as shown in the formula $v = \sqrt{\frac{2qV}{m}}$	1
			The mass of electrons increases due to relativistic effects meaning that engineers must use the relativistic mass (m) instead of the rest mass of electrons to produce the correct angle of deflection.	1
			If this were ignored, electrons would not be deflected to the correct angle and the images would be unintelligible.	1